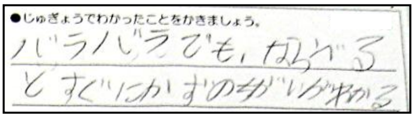
Model B: Representation of several aspects of the distribution, such as arranging in row or columns using a baseline	23

b. Taku's model

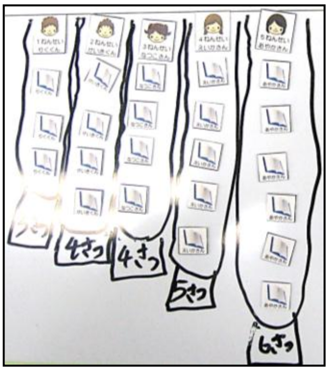


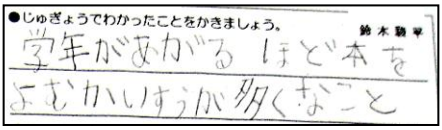
[Findings from the lesson]

Even when it is scattering, the difference in frequency is known shortly after arranging.

Model C: Representation of several aspects of the distribution with connections between aspects made	3

c. Shun's model





The older students have more chances of reading books.

Sharing classmates' models through demonstration and class discussion

In the latter half of the first lesson, the children shared two types of classmates' representations. The teacher's intention was to allow children to find the strengths and weaknesses of each representation. At first, the teacher asked Fusa to show his arrangement about data, where the book cards were grouped for each student card (*Model A*). Characteristics of the representation identified by other children were that, "We can understand well how many books were borrowed," and "It is arranging." Next, the teacher asked Tomo to present her arrangement in columns using a baseline (*Model B*). When the teacher asked about the difference between *Model A* and *Model B,* the child explained, "We can understand better the number (of books) Year 2 and Year 1 students borrowed, as the student cards are arranged in a row". When the teacher asked what the common feature of the two representations was, the child said, "Both are arranged straight." At the end of the first lesson, the children summarized the strength of each model as follows, "(About *Model A*,) a number is known immediately." and "(About *Model B*,) the difference in a number is known immediately."

In the first half of the second lesson, the teacher set a demonstration time for students to show their various representations, as the children had shared only two children's models in the first lesson. During this time, the children looked at classmates' representations with each other, as shown in Figure 3. The children found that "Classmates were writing the grid," and "Classmates were writing the numbers so that we could see how many books were borrowed."

Figure 3. Demonstration time

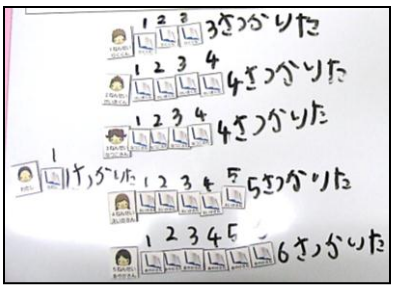
Children's Final Reasoning in Lesson 2

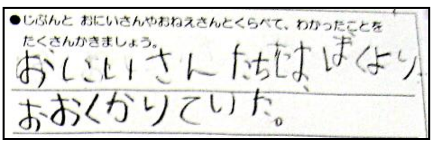
Table 2 shows how 27 children reasoned statistically when attempting the *Library Task* again in the second lesson, together with typical examples of children's models. All children added their own values, and could reflect upon their original models made in the first lesson by referring to the class discussion and the demonstration of classmates' representations.

Table 2. Children's models in lesson 2 (N=27)

Types of Models & Examples	Frequency
Model A: Representation of a single aspect of the distribution, such as grouping	0
Model B: Representation of several aspects of the distribution, such as arranging in row or columns using a baseline	20

a. Fusa's model

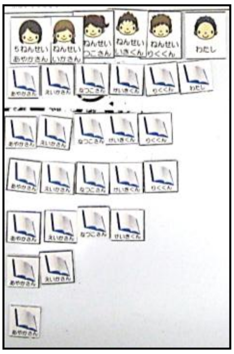


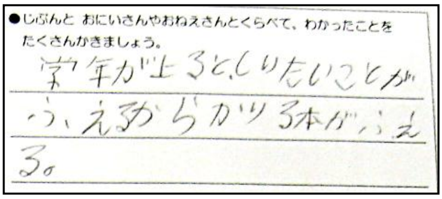


[Findings by comparing yourself to the older students]
The older students borrowed more books than I.

Model C: Representation of several aspects of the distribution with connections between aspects made	7

b. Taku's model





[Findings by comparing yourself to the older students]
The older students borrowed more books, as the more something to know increases.

No child made a model that represented a single aspect of the distribution (*Model A*). Twenty children made a model that represented several aspects of the distribution such as the frequency of books (*density*) and the difference of the frequency (*density*) (*Model B*). Seven children made a model that connected the elements of the distribution, such as the category of grade (*spread*) and the frequency of books (*density*) (*Model C*). One child, Fusa, developed his representation from *Model A* to *Model B*. Four children, including Taku, developed their representations from *Model B* to *Model C*. In *Model C,* three children could also connect patterns in the data with its context. For example, Taku inferred a reason for the functional relation between students' grades and the frequency of books, as indicated in his comment. He may be paying attention to the kind of borrowed books described in Figure 1. He also set a target about reading (by referring to his model that included his own value) as follows, "I want to borrow two or more books by the third term. Then, three or four, or more books." Thus some children could apply statistics to the context of daily life.

5. Discussion and implications

Although the reported findings are limited to a small sample, this paper has provided one example of how young children develop statistical reasoning about distributions from the viewpoint of model creating and sharing. The results in this paper indicate the possibility of practicing foundational teaching and learning related to distribution from the lower primary level.

In the first lesson, the majority of the children were able to pay attention to the elements of distribution, and moreover some children were able to connect them. The *Library Task*'s prompt to children to display representations using concrete objects might influence these results. Watson and Moritz (2001) indicated that even secondary students were not always aware of the need to display data in a way that allows counting. The awareness of others emphasized in the task can encourage children to create more sophisticated representations. Furthermore, the use of ordinal categorical data in the lesson might affect their representational sophistication. The printing of grades on student cards, as shown in Figure 1, and the provision of a data set where older students borrowed more books, might assist children to pay more attention to the order of the grades, and hence, frequency. However, not all the children perceived patterns in the data set, as shown in the difference between *Model B* and *Model C*. This highlights the gap between children's external and internal models related to distributions. Teachers need to assess both children's graphical representations and its statistical interpretation.

In the second lesson, the majority of the children maintained the representations arranging in row or columns using a baseline. The acceptance of the strength of the representation through the class discussion and the demonstration of representations might influence this result. However, some children added features such as a grid and/or vertical axis, although their representations were essentially the same as *Model B*. They could adapt the strengths of classmates' models into their own models, and give more consideration to the change or modification of each model. Several articles indicate sharing models when modelling is a rich learning experience for students (e.g., English, 2013; Kawakami and Matsuzaki, 2012). Model sharing in the class can enhance young children's statistical reasoning to a certain level.

Furthermore, during the second lesson a few children were able to connect the pattern of the distribution with its context. Some of them could apply the pattern to set their own targets about reading. They regarded the task setting as a concrete model connected with real life, and appreciated that data are "numbers with context" (Moore, 1990). Further research is needed to examine the extent to which a task setting including a data set should be simplified and symbolized so that young children can create statistical graphs from the task situation and apply them in the contextual domain.

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