A professional development project for teachers to learn how to teach problem-solving lessons through inquiry, exploration, discovery and extensions of mathematical concepts

Yuriko Yamamoto Baldin, Universidade Federal de São Carlos, Brazil

Introduction

Problem-solving based lessons are well recognized as effective teaching and learning strategies in School Mathematics, and the methodology of Problem Solving is explicitly recommended in National Curriculum Standards in Brazil (Brasil, 1998). Despite its importance, the courses of Teacher Education, either for pre-service or professional development of in-service teachers, lack efficiency as long as they remain as theoretic discussions and recommendations of practices, when they are not accompanied by the effective implementation of the methodology in actual classrooms. This paper presents an ongoing project of professional development for public school teachers (6^{th} to 9^{th} grades) in Brazil (Baldin et al., 2013), carried out since 2012. The main objective of this project is to overcome the noticed gap between intention and practice of many initiatives to improve Brazilian students' learning in mathematics.

Silver (2009) summarizes the recent interest of teacher educators, professional developers and researchers in "the design and facilitation of an approach to mathematics teacher education", known as practice-based professional development (PBPD). Its "conceptual heart" consists of "professional learning tasks (PLTs)" to be used "for and in" classroom practices. The main role of PLTs is to bridge the teacher practices to PBPD (Silver, 2009). The project discussed in this paper aligns to this research trend.

The difficulty of planning and implementing lessons based on the principles of problem-solving is a subject of research to develop modern and efficient curriculum design, as well as to educate teachers with contemporary vision of their role in mathematics classrooms, (Brasil, 1998), (Ball & Bass, 2003), (Stigler & Hiebert, 2009). The principles of problem solving are developed in the literature with major or minor differences in details. The simplest and most well-known by the teachers are those established by Polya (1945), through four steps: understanding the problem and its data; elaborating a strategy to solve; executing the solving plan; verifying/validating the solution and investigating the solving plan. These steps are meant to encourage students and problem-solvers "to learn to solve" a problem by oneself, but the crucial question is raised when it concerns *the teaching*: "What can be done to plan a problem-solving lesson to be performed in actual classrooms with large number of students?" Other questions follow subsequently, for instance:

"How is a problem-solving lesson special and different from exercises of application of the concepts/techniques just taught in a previous lesson?"

"Can a problem-solving lesson consists of a task of solving lists of problems selected from the adopted textbook?"

"Should a teacher be creative as to propose other problems (with what purpose in such case)?"

Many countries, Japan, Korea, China, Singapore, Costa Rica, to mention a few, have adopted problem-solving as the core of their curriculum to develop content knowledge, logical and deductive reasoning attitudes and competencies. The curriculum requires their citizens to be prepared for contemporary society. The examples of these countries show that understanding the importance of problem solving skills in Basic Education Schools is mandatory for pre and in service teachers.

A special challenge emerges for in-service teachers when they face the demand of introducing stimulating problems in classroom practices to promote organized mathematical reasoning and argumentation skills of students, in addition to mathematics concepts. Such problems are required to educate students to solve problems different from those familiar in textbooks. Creative problem solving teaching implies a paradigm shift to rethink the role of the teacher as mediator of students´ learning of problem solving during the lesson time.

This paper is organized in four parts: brief discussion of some structural difficulties of the Brazilian basic school system that motivated the design of the project; theoretical framework of the project; examples from the didactical material of the project; and concluding remarks.

Structural difficulties of Brazilian education system for teacher

Brazil, a continental size country, has been struggling to increase children's access to quality basic education. It has succeeded in enrolling more than 92% (~30.5 million) 6 to 14 year old children (1st to 9th grades), by 2011. The rapid increase in numbers in a short period of time represents a real challenge for established goals in the National Plan of Education, 2024, concerning the children's achievement in knowledge and competencies in reading and mathematics. Moreover, the education of mathematics teachers responsible for teaching the curriculum, especially for 6th to 9th grades, has not accompanied the growth in student numbers and the contemporary needs of quality education. The shortage of well-prepared mathematics teachers in all levels of basic education is a structural problem of the education system in Brazil. Annex 5 of UNESCO (2012) and the research of Isoda et al, (Chapter 21, 2012), point to structural difficulties that cause a gap between the teaching/learning perceptions of teachers from the elementary grades (1st to 5th grades, 6 to 10 years old) and intermediary grades (6th to 9th grades, 11 to 14 years old) of the Basic Education System in Brazil. This highlights deficiencies in the mathematics content knowledge of teachers, as well as in the methodologies of teaching and learning across the grades.

The deficiencies mentioned above have one cause: the fact that teachers for elementary grades are educated without specific content knowledge in basic mathematics, including arithmetic, geometry and organized thinking skills required in problem-solving and generalization process of progressive abstraction and representation language of mathematics. Education research in Brazil (for example Gatti, 2010) that compiled and analyzed many survey studies on the education of elementary school teachers in 2009-2010, distinguishes the deficient content course units in their curriculum, especially in mathematics. Teachers of elementary grades do not master the procedural processes of algorithms or mechanical activities necessary for any meaningful mathematical thinking in problem solving context.

On the other hand, the teacher education courses for intermediary level (6th to 9th grades) have a curriculum in which the knowledge of advanced mathematics is not connected to the school mathematics with its specific pedagogy, therefore teachers of these grades generally teach what they know about the content from the point of view of formalized mathematics. This results in an increase in difficulty for students, only a few of them follow the teachers´ explanation about problems, solutions and abstract concepts.

In this context, a research project on the design of an innovative professional development course for in-service teachers is necessary. Its main focus is to integrate diverse ingredients to improve teaching (content knowledge, pedagogy, and curriculum/school environment) and so promote meaningful learning for students. For this purpose, the project should aim at a paradigm shift of teachers´ attitude inside their classrooms. The strategy to attain this goal was to introduce challenging problems in lesson plans to promote a problem-solving approach in the classroom. Teachers should learn first how to stimulate the active participation of students in the solving steps, experimenting themselves in a renewed vision of mathematical thinking, to explore, discover, investigate, and extend the knowledge to next level, taking into account the mathematics knowledge and predicting their students' learning difficulties.

Therefore, the structure of the project was to design a PLT for in-service teachers, (Silver, 2009), working out the content knowledge through exploration of problems from textbooks not familiar to teachers, yet based on school curriculum content. The task posed to teachers was then to discover the core mathematics concepts present in the problem, explore the right approach according to the school grades, establish the meaning of the mathematical ideas developed through the steps of problem-solving, and extend the learned concepts to next level after studying the possible solving strategies under different perspectives. Considering the gaps in the formation of teachers as mentioned above, such tasks are challenging. The project was organized in hands-on workshops supported by on-line follow-ups to attend participants' difficulties in between workshop sessions. University researchers of Teacher Education courses coached the activities of participant teachers, who studied the problem-solving methodology to promote the participative learning of their students. The entire course was designed to be carried in four modules of 100 hours each. The project was called project PROF-OBMEP (Baldin et al, 2013).

The planning of PROF-OBMEP followed the "Lesson Study principles" (Isoda et al., 2007). It proposed activities to educate the teachers to help them first to understand the learning process of the students. They experience themselves the building of mathematical knowledge and develop skills through problem-solving approach. Lesson Study principles, lesson planning, classroom execution and reflection after lesson constituted the tasks of PROF-OBMEP as a PLT.

The theoretical framework

The basic framework of project PROF-OBMEP is the content knowledge for teachers (Shulman, 1986), established as the integrated triad of specific content knowledge, pedagogical knowledge and curricular context. In particular, the concept of pedagogical content knowledge indicates clearly the connection with the research on teaching and learning, and the cognitive research through studies of "*students' misconception and influences to subsequent learning"* (ibid p.10).

The systematized concept of practice-based professional development (PBPD) integrates mathematics content and mathematics pedagogy, advocated by (Ball & Bass, 2003), and mentioned in (Silver, 2009):

> *PBPD learning experiences are highly connected to and contextualized in professional practice settings, and advocates for this approach argue that this results in useful and usable knowledge that builds mathematics teachers' capacity for the kinds of complex, nuanced judgments required in mathematics teaching. (Ball & Bass, 2013).*

As already mentioned, the main focus of PROF-OBMEP is to provoke a paradigm shift of teachers' attitude inside the classroom, using problem solving as the methodology to change the learning process through student-centered activities. The change of teaching style from traditional teacher-centered explanation of solutions to the student-centered solving activities is the key methodology to achieve good learning results.

The traditional lesson style of problem solving in Brazil is the oral transmission by teachers of solution models. Sometimes the only solving way that teachers know lack further explorations of the conceptual meaning of mathematical ideas and the promotion of reasoning, central to problem-solving methodology of Polya, 1945. Therefore, the Project was designed to introduce teachers to the true meaning of the problem-solving steps, working out challenging problems from the Mathematics Olympiad for Public Schools - OBMEP (6^{th} to 12^{th} grades) (//www.obmep.org.br). The problems are studied before the actual resolution, analyzing their components, and *exploring the mathematical concepts* of the data, *discovering* new ideas and approaches to basic school curriculum knowledge, so that the resolution would entail further *investigation and extension to next level mathematical concepts.*

The proposed activities enhanced the pedagogical potential of *developing mathematical understanding through inquiries and extensions.* We find in (Isoda & Katagiri, 2012) an interesting parallel to the methods developed in the Project. This reference discloses the Katagiri's theory of the development of Mathematical Thinking that underlines the Problem –Solving Approach in Japanese Lesson Study. This theory is based on making adequate inquiries through and during the Problem-Solving steps, which assures the mathematical competencies. In the preface, Isoda asserts that "developing the theory of mathematical thinking with schoolteachers in the context of lesson study is also an innovation for mathematics education research, because it provides with the methodology as in reproductive science."

An Example illustrating the Pictorial Model to support extension of knowledge

Problem A (adapted from a problem of OBMEP 2014): 'A' went to a stationary shop and bought 3 notebooks in a promotional sale in which the second and third notebooks had respectively 20% and 30% discount on the price of the first notebook. The day after 'B' went to the shop and bought 3 notebooks the same type with no promotional price. Determine how much A paid less than B, in percentage.

The situation of this problem was explored using bar models, inspired by the Singapore Mathematics. The exercise resulted in a powerful instrument to enhance the mathematics concepts of the proportionality, as a relation part-whole in a problem of percentage. This brought the previous knowledge of fractional representation into a next level context, in which the understanding of the proportionality in relation to the whole unit of the context can be extended to general algebraic situation in other problems of proportionality, as will be indicated.

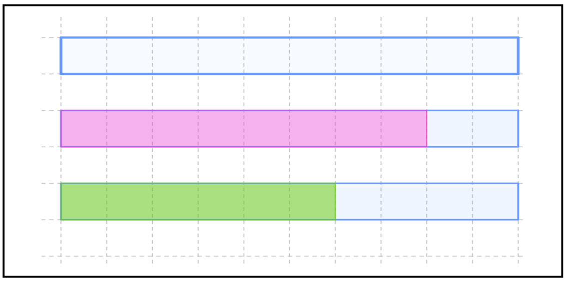

Figure 1: Representation of proportionality in the context of percentage.

The problem-solving steps were followed: interpreting first the meaning of the data then organizing it to figure out a strategy through inquiry. This allowed the teachers to recognize that they should identify the main concept first to work out the problem as the "comparison" of the total price paid by A with the "whole" paid by B. The pictorial representation of the discounts and prices helped with understanding the meanings of percentage in the price of each notebook. In this analysis, they learn the nature of the comparison that does not require the actual price of a notebook, as well as that the translation of such representation as fractions. This learning brings new understanding to the context of comparison (division as ratio, proportionality between two values). The strategy to consider 30 in the denominator of fractions to represent the context, meaning the total price paid by B, is achieved through asking adequate questions and discovery. Clearly other findings come out, in exploring different meanings that can be read in the pictorial setting.

This approach allows the discovery of different strategies to think about either the value of the total discount or the prices actually paid. Moreover, this problem permits the exercise of operations with fractions in many representations, as well as of equivalent fractions and their decimal representation, which in turn retrieves the meaning of percentage in the context.

Therefore, the fractional representation of a "part-whole" situation, learned in the elementary grades, helps to understand the meaning of proportionality as a ratio through "comparison" between "parts." As teachers notice that the price of a notebook is not a necessary data, they can explore the problem attributing any fixed value to the price of a notebook, and trace back the fractions worked in the solution by calculating them and checking the percentage of their previous answer. In actual practice, this problem was enriched by the use of problem solving approach with diverse strategies, even the one that does not use any calculation but only recognizing by visualization in Bar Model the ratio between the prices paid by A and B.

The pictorial representation of a problem to support mathematical thinking implies further extensions facilitating the work with more challenging problems usually avoided in ordinary lessons, as in the following example, considered at first sight very difficult by teachers but made possible with the extension of the concept of proportionality in an algebraic approach, and helped by pictorial representation:

Problem B (adapted from OBMEP 2014): Two friends A and B took a taxi ride agreeing to share the fare proportionally to the distance run by each one. The ride started with a fixed value of $4.00 and A got off when the price was $28.00, without paying. At the end of ride B paid the total amount of $44.00.How much must A pay to B?

Conclusion

One key activity that permeated the workshops of the project was the connection of the mathematics content of each problem with competencies stated in the national curriculum standards (Brasil, 1998). This feature of the project along with the discovery of the continuous process of development and extension of mathematical ideas across the school grades, make a breaking point for resistance to new methodologies in the professional development of Brazilian mathematics teachers. The project PROF-OBMEP is one initiative to bring hope to improve mathematics lessons of Brazilian schools. The success of the project should be conferred by exterior assessment of students' achievement in the classrooms of their participants consistently in the following years.

References

Baldin, Y. Y., Silva. A. F., Fürkotter, M., Rodrigues, J. C. & Bezerra, D. B. (2013). *Texto do Projeto PROF-OBMEP (Text of the Project PROF-OBMEP)*, IMPA, Rio de Janeiro. (in Portuguese).

Ball, D. L. & Bass, H. (2003). Toward a practice-based theory of mathematics knowledge for teaching. In B. Davis & E. Simmt (Eds.), *Proceedings of the 2002 annual meeting of the Canadian Mathematics Education Study Group (CMESG),* Edmonton: CMESG.

Brasil. (1998). *Parâmetros Curriculares Nacionais: Matemática (National Curriculum Standards: Mathematics),* Ministry of Education, Brasilia.

Gatti, B. (2010). Formação de professors no Brasil: características e problemas (The formation of teachers in Brasil: characteristics and issues), *Educ. Soc. Campinas, v.31*, n.113, pp 1355-1379, Campinas. (in Portuguese).

Isoda, M., Arcavi, A. & Mena L. A. (Eds) (2012), *El Estudio de Clases Japonés en Matemáticas*, Ediciones Universitárias de Valparaiso, 3rd edition, Valparaiso. (in Spanish).

Isoda, M., Stephens, M., Ohara, Y. & Miyakawa, T. (2007). *Japanese Lesson Study in Mathematics: Its Impact, Diversity and Potential for Educational Improvement*. Singapore: World Scientific.

Isoda, M. & Katagiri, S. (2012). *Mathematical Thinking: How to Develop it in the Classroom.* Singapore: World Scientific.

Polya, G. (1945). *How to solve it?* New Jersey: Princeton University Press.

Silver, E. A. (2009). Toward a More Complete Understanding of Practice-Based Professional Development for Mathematics Teachers. In R. Even & D. L. Ball

(Eds.). *The Professional Education and Development of Teachers, The 15th ICMI Study*. New York: Springer.
Stigler, J. & Hiebert, J. (1999). *The Teaching Gap: Best Ideas from the World's Teachers for Improving Education in the Classroom* New York: Free Press.
Shulman, L. (1986). Those Who Understand: Knowledge Growth in Teaching, *Educational Researcher*, *Vol 15*, n. 2, pp. 4-14.
UNESCO (2012), *Les défis de l'enseignement des mathématiques dans l'education de base*. Paris: UNESCO.

Acknowledgement
The author acknowledges the IMPA-OBMEP/CAPES for financial support of the project PROF-OBMEP from 2012–2014, and colleagues of this project, Aparecida F Silva, Monica Fürkotter, José C Rodrigues and Debora Bezerra. Also many thanks are due to all teacher participants from public schools and university partners in the execution of the project, for their enthusiasm and collaboration to take the activities to their classrooms.

Yuriko Yamamoto Baldin
Universidade Federal de São Carlos, São Carlos, SP, Brasil
yuriko@dm.ufscar.br