A multi-strand framework for assessing Year Four pupils' proficiency in measurement formulae

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Background of the study

Measurement formulae such as perimeter, area and volume formulae are general relationships that produce measurements when values are specified for the variables in the formulae. The study of measurement formulae is important in the mathematics curriculum because it offers an opportunity for learning and applying other mathematical concepts and skills such as number operations, geometrical ideas, statistical concepts, and notions of function. The study of measurement formulae also emphasizes connections within mathematics and between mathematics and areas outside of mathematics such as science and art. It is also important because of the practicality and pervasiveness of measurement formulae in so many aspects of everyday life (NCTM, 2000).

In view of its importance, the study of measurement formulae forms an important part of the Malaysian primary school mathematics curriculum. In the Year Four Mathematics Curriculum and Assessment Standard Document of the Primary School Standard Curriculum, Year Four pupils begin to learn how to find the perimeter of rectangles, squares, triangles, and polygons. They also learn how to calculate the area of rectangles, squares, and triangles using square grid and formulae. Further, they learn how to calculate the volume of cubes and cuboids using 1 cm^3 unit cubes and formulae (Malaysian Ministry of Education, 2013). Next, in Year Five, primary pupils learn how to calculate the perimeter and area of composite two-dimensional shapes involving squares, rectangles, and triangles. They also learn how to solve problems involving perimeters of composite two-dimensional shapes. In addition, they also learn how to calculate the volume of composite three-dimensional shapes involving cubes and cuboids as well as to solve problems involving volume of composite three-dimensional shapes (Malaysian Ministry of Education, 2006a). Finally, in Year Six, primary pupils learn how to calculate the perimeter and area of composite two-dimensional shapes involving two or more quadrilaterals and triangles as well as to solve problems in real context involving perimeter and area of two-dimensional shapes. They also learn how to calculate the surface area and volume of composite three-dimensional shapes involving cubes and cuboids as well as to solve problems involving surface area and volume of composite three-dimensional shapes (Malaysian Ministry of Education, 2006b).

As they go from Year 4 to Year 6, all primary pupils should become increasingly proficient in measurement formulae. Proficiency in measurement formulae should enable them to continue their study of measurement in particular and mathematics in general in secondary school and beyond. Proficiency in measurement formulae should also enable them to cope with the mathematical challenges of everyday life (National Research Council, 2001). In Malaysia, Year Six pupils' proficiency in measurement formulae is assessed in the Mathematics papers of the Primary School Achievement Test, which is a national examination taken by all primary pupils at the end of their sixth year in primary school before they leave for secondary school.

But, in the Mathematics papers of the Primary School Achievement Test, the Malaysian Examinations Syndicate reported that Year Six pupils made various types of mistakes in answering questions involving measurement formulae. For example, the common mistakes made by Year Six pupils in calculating the perimeter of a whole diagram consisting of three congruent right-angled triangles with two of the triangles having a common hypotenuse to form a rectangle were: (i) adding the common hypotenuse which is located inside the diagram; or (ii) calculating the perimeter of one right-angled triangle and then multiplying the perimeter by three (Malaysian Malaysian Examinations Syndicate, 2005). In calculating the perimeter of a whole diagram consisting of a right-angled triangle and a rectangle, the common mistakes made by Year Six pupils were: (i) inability to find the length of sides without given measurement; or (ii) adding the length of the side which is located inside the diagram (Malaysian Malaysian Examinations Syndicate, 2008). Further, the common mistakes in calculating the perimeter of a shaded region were adding the length of all the sides or some of the sides of the given diagram containing the shaded region instead of adding the length of all the sides of the shaded region (Malaysian Malaysian Examinations Syndicate, 2010). In calculating the area of a shaded region consisting of two right-angled triangles, the common mistakes were: (i) calculating the area of a triangle using the area of a rectangle; (ii) calculating the area of one of the two right-angled triangles; or (iii) calculating the perimeter of the shaded region (Malaysian Malaysian Examinations Syndicate, 2007). In calculating the volume of a cube with edges 6 cm long, the common mistakes were: (i) adding the length of 6 edges (that is, 6 + 6 + 6 + 6 + 6 + 6); or (ii) multiplying the length of 2 edges (that is, 6 x 6) (Malaysian Malaysian Examinations Syndicate, 2010). These reports indicate that in general Malaysian Year Six pupils' proficiency in measurement formulae is unsatisfactory.

In addition, in the latest Trends in International Mathematics and Science Study 2011 (TIMSS 2011), Malaysian Form Two students' performance in the four TIMSS released items involving measurement formulae was unsatisfactory. For the first released item (ID_M052084) on calculating the area of a square with a given perimeter of 36 cm, only 40% of Malaysian Form Two students were able to answer it correctly. As a result, Malaysian Form Two students' performance was ranked 27th and the percent correct of Malaysian Form Two students was significantly lower than the international average of 47%. For the second released item (ID_M042201) on finding the length of a rectangular box with a given volume of 200 cm^3, only 42% of Malaysian Form Two students were able to answer it correctly. Consequently, Malaysian Form Two students' performance was ranked 23rd and the percent correct of Malaysian Form Two students was slightly lower than the international average of 43%. For the third released item (ID_M032116) on finding the perimeter of a square with a given area of 144 cm^2, only 43% of Malaysian Form Two students were able to answer it correctly. As a result, Malaysian Form Two students' performance was ranked 25th and the percent correct of Malaysian Form Two students was slightly lower than the international average of 45%. For the fourth released item (ID_M032623) on finding the area of a shaded region in cm^2 using the area of rectangle minus the area of right-angled triangle, only 29% of Malaysian Form Two students were able to answer it correctly. Consequently, Malaysian Form Two students' performance was ranked 23rd and the percent correct of Malaysian Form Two students was significantly lower than the international average of 36% (Foy, Arora & Stanco, 2013). These findings indicate that in general Malaysian Form Two students' proficiency in measurement formulae is unsatisfactory.

Statement of the problem

Malaysia aspires to be ranked in the top third of countries in terms of mathematics performance in TIMSS within 15 years (Malaysian Ministry of Education, 2012). One way of achieving this aspiration is to produce pupils who are mathematically proficient in measurement formulae starting from the earliest year of schooling in which they begin to learn the formulae, that is Year 4. In general, mathematical proficiency as defined by the National Research Council (2001) has five intertwined components or strands namely conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. These five strands provide a multi-strand framework for assessing pupils' knowledge, skills, abilities, and beliefs that constitute mathematical proficiency (National Research Council, 2001).

Yet, to date there is no available framework and test for assessing Year 4 pupils' proficiency in measurement formulae in Malaysia that takes into account all the five strands of mathematical proficiency as defined by the National Research Council (2001). Hence, there is an urgent need to develop a multi-strand framework and test for assessing Year 4 pupils' proficiency in measurement formulae so that appropriate diagnosis and intervention programs can be provided by the teachers to improve their pupils' performance in each of the five strands of proficiency in measurement formulae starting from the earliest schooling year of learning the formulae, that is Year 4.

Purpose of the study

This research project aims to address these concerns by developing a multi-strand framework and test for assessing Year Four pupils' proficiency in measurement formulae which involves two phases: (1) development of a multi-strand framework, and (2) development of a multi-strand test based on the multi-strand framework. At this stage, this paper discusses the first phase of the project which is the development of a multi-strand framework for assessing Year Four pupils' proficiency in measurement formulae based on the mathematical proficiency framework of the National Research Council (2001) and the Malaysian Year Four Mathematics Curriculum and Assessment Standard Document of the Primary School Standard Curriculum (Malaysian Ministry of Education, 2013).

Mathematical proficiency framework

According to the National Research Council (2001), mathematical proficiency comprises five intertwined components or strands: (1) *conceptual understanding* - comprehension of mathematical concepts, operations and relations; (2) *procedural fluency* - skill in carrying out procedures flexibly, accurately, efficiently and appropriately; (3) *strategic competence* - ability to formulate, represent and solve mathematical problems; (4) *adaptive reasoning* - capacity for logical thought, reflection, explanation and justification, and; (5) *productive disposition* - habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy (p. 116).

Year 4 pupils' proficiency in measurement formulae framework

Based on the mathematical proficiency framework of the National Research Council (2001), Year Four pupils' proficiency in measurement formulae also comprises five intertwined components or strands: (1) *conceptual understanding* - comprehension of measurement formulae; (2) *procedural fluency* - skill in carrying out measurement formulae procedures flexibly, accurately, efficiently and appropriately; (3) *strategic competence* - ability to formulate, represent and solve mathematical problems involving measurement formulae; (4) *adaptive reasoning* - capacity for logical thought, reflection, explanation and justification of solutions to problems involving measurement formulae, and; (5) *productive disposition* - habitual inclination to see measurement formulae as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy (p. 116). The five strands are interwoven and interdependent in the development of proficiency in measurement formulae. Each strand is explained in more detail below.

Conceptual understanding

Year Four pupils with conceptual understanding should have an integrated and functional grasp of measurement formulae, know more than isolated measurement formulae, understand why measurement formulae are important and the kinds of contexts in which they are useful, organize their knowledge of measurement formulae into a coherent whole enabling them to learn new measurement formulae by connecting those measurement formulae to what they already know (Bransford, Brown, & Cocking, 1999; Carpenter & Lehrer, 1999; Greeno, Pearson, & Schoenfeld, 1997; Hiebert, 1986; Hiebert & Carpenter, 1992). Conceptual understanding helps Year Four pupils in retention because measurement formulae learned with understanding are connected, easier to remember and use, and can be reconstructed when forgotten (Hiebert & Carpenter, 1992; Hiebert & Wearne, 1996). It also provides the basis for solving new and unfamiliar problems (Bransford, Brown, & Cocking, 1999). When pupils have acquired conceptual understanding, they gain confidence which then provides a base from which they can move to another level of understanding (National Research Council, 2001).

Procedural fluency

Year Four pupils with procedural fluency have knowledge of measurement formulae procedures, knowledge of when and how to use them appropriately as well as skills in performing them flexibly, accurately, and efficiently. Procedural fluency and conceptual understanding are interwoven because understanding makes learning skills easier, less susceptible to common errors, and less prone to forgetting. Likewise, a certain level of skill is required to learn many measurement formulae with understanding, and using measurement formulae procedures can help strengthen and develop that understanding (National Research Council, 2001). In an experimental study conducted by Pesek and Kirshner (2000), fifth-grade pupils who first received instruction on procedures for calculating area and perimeter followed by instruction on understanding those procedures did not perform as well as pupils who received instruction focused only on understanding. This is because when pupils practice procedures without understanding, they are learned as isolated bits of knowledge and there is a danger they will practice incorrect procedures, thereby making it more difficult to learn correct ones. Thus, initial learning with understanding can make learning more efficient (National Research Council, 2001).

Strategic competence
Year Four pupils with strategic competence have the ability to formulate mathematical problems involving measurement formulae, represent them numerically, symbolically, verbally or graphically, and solve them. In addition, they should know a variety of solution strategies and which strategies might be useful for solving a specific problem involving measurement formulae. Flexibility is a fundamental characteristic needed throughout the problem-solving process and it develops through the broadening of knowledge required for solving non-routine problems involving measurement formulae rather than just routine problems (National Research Council, 2001). According to Pólya (1945), routine problems are problems that pupils know how to solve based on past experience. They know a correct solution method and are able to apply it when solving a routine problem. Thus, routine problems require reproductive thinking, that is pupils need only to reproduce and apply a known solution procedure. But, non-routine problems are problems for which pupils do not immediately know a usable solution method. In other words, non-routine problems require productive thinking because pupils need to invent a way to understand and solve the problem (National Research Council, 2001).

Adaptive reasoning
Year Four pupils with adaptive reasoning have the capacity for logical thought, reflection, explanation and justification of solutions to problems involving measurement formulae (National Research Council, 2001). According to Alexander, White, and Daugherty (1997), pupils are able to display reasoning ability when (a) they have a sufficient knowledge base; (b) the task is understandable and motivating, and; (c) the context is familiar and comfortable. According to the National Research Council (2001), the ability to justify or provide sufficient reason for one's work or solution to non-routine problems involving measurement formulae is one manifestation of adaptive reasoning.

Productive disposition
Year Four pupils with productive disposition have the tendency to see measurement formulae as sensible, useful and worthwhile, to believe that steady effort in learning measurement formulae pays off, to see oneself as an effective learner and doer of mathematics involving measurement formulae and are confident in their knowledge and ability. Year Four pupils' productive disposition develops when the other strands do and helps each of them develop (National Research Council, 2001). Cross-cultural research studies (e.g., Stevenson & Stigler, 1992; Bempechat & Drago-Severson, 1999) have found that U.S. children are more likely to attribute success in school to ability rather than effort when compared with students in East Asian countries.

In sum, Year Four pupils who are proficient with measurement formulae are able to understand, compute, solve, reason, and possess a productive disposition toward measurement formulae. They believe that measurement formulae should make sense, that they can solve mathematical problems involving measurement formulae by working hard on them, and that becoming proficient with measurement formulae is worth the effort (National Research Council, 2001).

Malaysian Year Four Mathematics Curriculum and Assessment Standard Document

In the Malaysian Year Four Mathematics Curriculum and Assessment Standard Document of the Primary School Standard Curriculum (Malaysian Ministry of Education, 2013), the mathematics contents comprise four learning areas, namely Numbers and Operations, Measurement and Geometry, Relationships and Algebra, and Statistics and Probability. The learning area of Measurement and Geometry consists of five topics, that is Time, Length, Mass, Volume of Liquid, and Space. The topic of Space is divided into four subtopics: Angles, Parallel and Perpendicular Lines, Perimeter and Area, and Volume of Solids. Year Four pupils learn perimeter, area and volume formulae in the subtopics of Perimeter and Area, and Volume of Solids, respectively. Table 1 summarizes the Content Standards, Learning Standards, Mastery Level and Performance Standards for the topic of Space in the Malaysian Year Four Mathematics Curriculum and Assessment Standard Document of the Primary School Standard Curriculum (Malaysian Ministry of Education, 2013).

Table 1. Content Standards, Learning Standards, Mastery Level and Performance Standards for the Topic of Space

<table>
<tr><th>Content Standards</th><th>Learning Standards</th><th>Mastery Level</th><th>Performance Standards</th></tr>
<tr><td rowspan="6">1. Angles

2.Parallel and Perpendicular Lines

3.Perimeter and area

4.Volume of Solids</td><td rowspan="6">Recognize and name right angles, acute angles and obtuse angles in rectangles, squares, and triangles

Recognize and name:
i) parallel lines,
ii) perpendicular lines,
in two-dimensional shapes

a) Determine the perimeter of rectangles, squares, triangles and polygons
b) Determine the area of rectangles, squares and triangles using square grid and formulae

Determine the volume of cubes and cuboids using 1 cm^3 unit cubes and formulae</td><td>1</td><td>State the types of angles and lines in two-dimensional shapes</td></tr>
<tr><td>2</td><td>Explain the meaning of perimeter, area and volume using formulae</td></tr>
<tr><td>3</td><td>Calculate perimeter, area and volume</td></tr>
<tr><td>4</td><td>Solve routine problems in everyday life involving angles, lines, perimeter, area and volume</td></tr>
<tr><td>5</td><td>Solve routine problems in everyday life involving angles, lines, perimeter, area and volume using various strategies</td></tr>
<tr><td>6</td><td>Solve non-routine problems in everyday life involving angles, lines, perimeter, area and volume creatively and innovatively</td></tr>
</table>

Source: Malaysian Ministry of Education (2013, p.43)

A multi-strand framework for assessing Year 4 pupils' proficiency in measurement formulae

Based on the mathematical proficiency framework of the National Research Council (2001) and the Malaysian Year Four Mathematics Curriculum and Assessment Standard Document of the Primary School Standard Curriculum (Malaysian Ministry of Education, 2013), the multi-strand framework for assessing Year Four pupils' proficiency in measurement formulae is explained in more detail in Table 2. The first column explains the meaning of the five strands of Year Four pupils' proficiency in measurement formulae. The second column explains the meaning of the five strands of Year Four pupils' proficiency in perimeter formula of a rectangle, square, triangle and polygon. The third column explains the meaning of the five strands of Year Four pupils' proficiency in area formula of a rectangle, square, triangle and polygon. Lastly, the fourth column explains the meaning of the five strands of Year Four pupils' proficiency in volume formula of a cube and cuboid.

Table 2. A multi-strand framework for assessing Year Four Pupils' proficiency in measurement formulae

Strand	Perimeter	Area	Volume
1. Conceptual understanding: Comprehension of measurement formulae	Comprehension of perimeter formula of i) a rectangle, ii) a square, iii) a triangle, and iv) a polygon	Comprehension of area formula of i) a rectangle, ii) a square, and iii) a triangle	Comprehension of volume formula of i) a cuboid ii) a cube
2. Procedural fluency: Skill in carrying out measurement formulae procedures flexibly, accurately, efficiently and appropriately	Skill in carrying out perimeter formula procedure of i) a rectangle, ii) a square, iii) a triangle, and iv) a polygon flexibly, accurately, efficiently and appropriately	Skill in carrying out area formula procedure of i) a rectangle, ii) a square, and iii) a triangle flexibly, accurately, efficiently and appropriately	Skill in carrying out volume formula procedure of i) a cuboid ii) a cube flexibly, accurately, efficiently and appropriately
3. Strategic competence: Ability to formulate, represent and solve mathematical problems involving measurement formulae	Ability to formulate, represent and solve mathematical problems involving perimeter formula of i) a rectangle, ii) a square, iii) a triangle, and iv) a polygon	Ability to formulate, represent and solve mathematical problems involving area formula of i) a rectangle, ii) a square, and iii) a triangle	Ability to formulate, represent and solve mathematical problems involving volume formula of i) a cuboid ii) a cube
4. Adaptive reasoning:	Capacity for logical thought, reflection,	Capacity for logical thought, reflection,	Capacity for logical thought,

Capacity for logical thought, reflection, explanation and justification of solutions to problems involving measurement formulae	explanation and justification of solutions to problems involving perimeter formula of i) a rectangle, ii) a square, iii) a triangle, and iv) a polygon	explanation and justification of solutions to problems involving area formula of i) a rectangle, ii) a square, and iii) a triangle	reflection, explanation and justification of solutions to problems involving volume formula of i) a cuboid ii) a cube
5. Productive disposition: Habitual inclination to see measurement formulae as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy	Habitual inclination to see perimeter formula of i) a rectangle, ii) a square, iii) a triangle, and iv) a polygon as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy	Habitual inclination to see area formula of i) a rectangle, ii) a square, and iii) a triangle as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy	Habitual inclination to see volume formula of i) a cuboid ii) a cube as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy

Conclusion

While the Malaysian Year Four Mathematics Curriculum and Assessment Standard Document provides general descriptions of the mastery levels for all the sub-topics of Space, the multi-strand framework provides specific descriptions of the five proficiency strands for each of the measurement formulae. First, the Performance Standard 'Explain the meaning of perimeter, area and volume using formulae' in the Standard Document provides a general description of Mastery Level 2 for the subtopics of perimeter and area as well as volume of solids. But, the first strand of proficiency in the multi-strand framework provides specific descriptions of conceptual understanding of perimeter and area formulae of a rectangle, square, triangle, and polygon and also for volume formula of a cuboid and cube. Second, the Performance Standard 'Calculate perimeter, area and volume' in the Standard Document provides a general description of Mastery Level 3 for the subtopics of perimeter and area as well as volume of solids. However, the second strand of proficiency in the multi-strand framework provides specific descriptions of procedural fluency with perimeter and area formulae of a rectangle, square, triangle and polygon and also for volume formula of a cuboid and cube. Third, the Performance Standard 'Solve routine problems in everyday life involving angles, lines, perimeter, area and volume' in the Standard Document provides a general description of Mastery Level 4 for all the subtopics of Space. But, the third strand of proficiency in the multi-strand framework provides specific descriptions of strategic competence in perimeter and area formulae of a rectangle, square, triangle and polygon and also for volume formula of a cuboid and cube. Fourth, while the Performance Standard 'Solve routine problems in everyday life involving angles, lines, perimeter, area and volume using various strategies' in the Standard Document provides a general description of Mastery

Level 5 for all the subtopics of Space, the fourth strand of proficiency in the multi-strand framework provides specific descriptions of adaptive reasoning in perimeter and area formulae of a rectangle, square, triangle and polygon and also for volume formula of a cuboid and cube. Lastly, the Performance Standard 'Solve non-routine problems in everyday life involving angles, lines, perimeter, area and volume creatively and innovatively' in the Standard Document provides a general description of Mastery Level 6 for all the subtopics of Space, whereas the fifth strand of proficiency in the multi-strand framework provides specific descriptions of productive disposition toward perimeter and area formulae of a rectangle, square, triangle and polygon and also for volume formula of a cuboid and cube.

In conclusion, the multi-strand framework provides more specific and fine-grained descriptions of the Year Four pupils' proficiency in each of the measurement formulae of two- and three-dimensional shapes. Thus, after a panel of experienced primary school mathematics teachers has validated the multi-strand framework, it will be used in the second phase of the research project to develop a multi-strand test for assessing Year Four pupils' proficiency in each of the measurement formulae of two- and three-dimensional shapes. It is hoped that the multi-strand framework and test will be able to provide a more appropriate diagnosis of Year Four pupils' proficiency in measurement formulae so that effective intervention programs could be provided by the teachers to improve their pupils' performance in each of the five strands of proficiency in measurement formulae of two- and three-dimensional shapes starting from the earliest schooling year of learning the formulae.

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