How teachers should instruct and represent mathematical ideas to enhance student engagement: Viewpoints of elementary school students

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Introduction

Student disengagement has been an immediate and persisting issue in the literature. Studies have reported that engaging students in mathematics is more difficult compared with other subjects (Kong, Wong, & Lam, 2003). The level of student engagement influences their development of mathematical literacy, including their achievements, abilities, and appreciation of this subject (Attard, 2012; Bodovski & Farkas, 2007; Park, 2005). In modern society, mathematical literacy is crucial for everyone (OECD, 2013). Thus, exploring how to engage students in mathematics is critical.

Previous studies have revealed that distinct levels of engagement are associated with distinct teaching instructions (Bodovski & Farkas, 2007); this phenomenon necessitates exploring information specific to the methods teachers must adopt to increase students' engagement in mathematics. Numerous studies have focused on investigating the type of student activities that can enhance and maintain engagement. Previous studies have also reported that hands-on activities, small group discussions, student–teacher discourses, and embedding technology group works contributed to students' engagement in mathematics (Blumenfeld & Meece, 1988; Cavanagh, 2011; Park, 2005). In addition to student activities, traditional lecturing still plays a crucial role in mathematics classes and also referred to by students for contributing to facilitating and maintaining engagement. Students in Cavanagh's study described that their engagement was enhanced if their teacher delivered concepts in a helpful and meaningful manner. Two dimensions of how a teacher can deliver mathematical ideas are in this manner. The first involves how teachers instruct ideas, and the second involves how teachers represent ideas (Piccolo, Harbaugh, Carter, Capraro, & Capraro, 2008).

Following the global trend of mathematics education, Taiwan has adopted student-centered views from the West to reform the curriculum of mathematics and to improve mathematical teaching (Lin & Li, 2009). Hence, we believe that the voice of students should be considered seriously. Therefore, the present study collected the perspectives of students in Taiwan to explore the research questions. The objectives of this study were (a) investigating the perceptions of elementary school students regarding how teachers should instruct and represent mathematical ideas, and (b) investigating their perspectives on how these teaching behaviors are related with student engagement.

Research method

Conceptual framework

Engagement

Newmann (1992) indicated that engagement represented "active involvement, commitment, and concentrated attention, in contrast to superficial participation, apathy, or lack of interest." Marks (2000) conceptualized student engagement in academic work as "a psychological process, specifically, the attention, interest, investment, and effort students expend in the work of learning." These descriptions reflect the multifaceted construct of engagement. In the literature, student engagement is often explored in three manners: behavioral engagement, affective engagement, and cognitive engagement (Fredricks, Blumenfeld, & Paris, 2004). Regarding the subject of mathematics, behavioral engagement draws on the ideas of sustained participation in and concentration on learning activities (Attard, 2012). In the literature, realizing concepts as well as thinking about and solving problems are critical mental activities to determine whether students are cognitively engaged in mathematics classes (Gresalfi & Barab, 2011). Affective engagement is related to motivations to learn mathematics (Fredricks et al., 2004).

Associations between student engagement and teachers' instruction and representation of mathematical ideas

Regarding how teachers should instruct mathematical ideas to enhance engagement, students have mentioned the contribution from teachers' explainations in a well-spoken, clear, and concise manner (Cavanagh, 2011). Scaffolding is another approach of instructing mathematical ideas to help engage students; this approach involves teaching processes that start from simple tasks or from basic concepts (Henningsen & Stein, 1997). In addition, studies have reported that students have a high-level of engagement when they can generate connections among ideas. Student engagement increases if teachers instruct mathematical ideas by linking the ideas with the students' prior knowledge (Bennett & Desforges, 1988). Henningsen and Stein (1997) indicated that teachers could enhance engagement by frequently drawing connections and syntheses among various mathematical ideas. Previous studies have also reported that generating links to the real world affects engagement. Using real-life examples helps engage students (Gresalfi & Barab, 2011; Park, 2005).

Using representations is integral and indispensable for teachers when conveying mathematical ideas (Stylianou, 2010). Mathematics involves various types of representation, including graphical displays, numerical examples, symbolic expression, real-life examples, and so on (Piccolo et al., 2008; Stylianou, 2010). Each representation has specific strengths and shows as well as hides some facets of a mathematical idea (Kaput, 1992). Various uses of representations influence students' understanding of mathematical ideas, thus affecting their engagement (Kawanak & Stigler, 1999).

This study constructed a conceptual framework (Figure 1) based on the foregoing discussion.

Design and instruments

The questionnaire administered in this study was adapted from Hsieh (2009), which was used to investigate secondary school students' opinions regarding what an ideal mathematics teacher should do when teaching. Hsieh conducted a qualitative pilot study, using open-ended questions, on 238 high school students. A content analysis on the students' responses and literature review were performed to obtain dimensions and items for a questionnaire that comprised dichotomous items (Hsieh & Wang, 2014). The current study adapted Hsieh's questionnaire to fit the situations of the elementary school level and surveyed elementary school students. Students were asked, through dichotomous items, to state whether a good elementary school teacher would perform a certain teaching behavior for some scenarios.

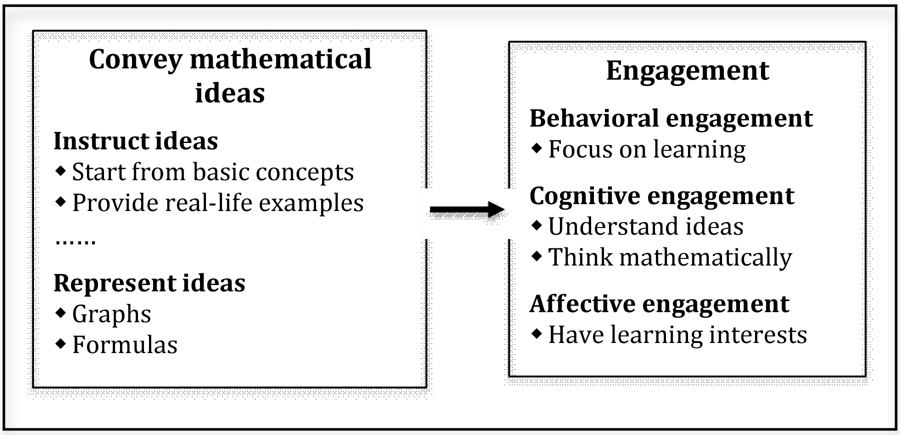

Figure 1. The study framework

In this study, three questions in the questionnaire. One question was related to whether a good elementary school teacher would engage students on this facet when teaching mathematics; 4 items were included. The other two questions were related to whether a good elementary school teacher would perform a certain teaching behavior when conveying mathematical ideas. One of the remaining two questions concerned instructing mathematical ideas, and it comprised 6 items; the other question was related to representing mathematical ideas, and it comprised 7 items. Figure 2 shows an example of one of the questions; Tables 1 and 2 show complete lists of the questions.

When teaching us mathematical ideas, a very good elementary school teacher would…

☐1. Provide many examples from life when teaching ideas

……

☐4. Start from basic concepts so that we can have a full picture of the concept

……

Figure 2. The question related to instructing mathematical ideas

Participants

The sample comprised 2102 Taiwanese elementary school students in 78 classes from 26 schools spread through 25 cities in Taiwan. The sampled schools were randomly selected, and in each of these schools, one fourth, fifth, and sixth grade class were chosen randomly.

Data analysis

Regarding research purpose (a), this study performed an exploratory factor analysis (EFA) with oblique rotation to determine the factor structures of the scales of instructing mathematical ideas and representing mathematical ideas, respectively. In this study, the EFA was conducted with M-plus 6.12 by using a robust weighted least squares estimator that is typically considered robust to non-normality (Flora & Curran, 2004). To determine the number of latent factors extracted, this study considered several criteria, including a comparative fit index (CFI) greater than 0.90, a Tucker–Lewis Index (TLI) higher than 0.90, and a root mean square error of approximation (RMSEA) lower than 0.08 (Fan, Thompson, & Wang, 1999; Hu & Bentler, 1999; Kline, 2011). The number of the eigenvalues greater than 1 were also examined in this study according to the Kaiser–Guttman rule. The structures identified through EFA were subsequently tested through a confirmatory factor analysis process. Descriptive information, including the percentage of checking for each item and weighted average percentage of checking for each latent factor, were also computed using the factor loadings of the indicators as weights.

To meet the second objective of this study, the structural relationships between the latent constructs were examined by conducting a series of structural equation models (SEM). The relationships between instructing mathematical ideas and engagement as well as those between representing mathematical ideas and engagement were tested. Model fit statistics, including CFI, TLI, and RMSEA, were used as previously mentioned.

Results

Relationships between instructing mathematical ideas and engagement

Table 1 shows the students' opinions regarding the teaching behaviors elementary school teachers should adopt when instructing mathematical ideas; this table also shows the EFA results. Two factors, each of which comprised 3 items, captured instructing mathematical ideas. The first factor was denoted "reasoning," and it represents instructing students by helping them understand the meaning of the mathematical ideas. The second factor was denoted "connection," and it represents instructing students by helping them establish connections among the ideas.

Table 1. Descriptive information and pattern coefficients of instructing math ideas

Items	Pattern coefficients		Checking percentage
	Factor 1	Factor 2	
Illustrate math ideas clearly	0.959		94%
Start from basic concepts	0.490		92%
Apply simple computations to introduce formulas	0.481		92%
Compare new math ideas with others		0.676	76%
Provide examples from life		0.641	91%
Extend from math ideas already learned		0.580	94%

The weighted average percentages of "reasoning" and "connection" were 93% and 88%, respectively. These high percentages imply that students perceived both factors as the aspects a good teacher must focus on; "reasoning" registered a higher endorsement

(p < .01, obtained by conducting a pair-sample t test) than "connection." The checking percentages of all items were higher than 90%, except "comparing new math ideas with others," which was endorsed by only 76% of students. From the standpoint of "to be a good teacher," the results indicated that all 3 methods regarding "reasoning" and two methods embedded in "connection" (helping students connect the mathematical ideas to real-life and prior knowledge) were indispensable baselines for instructing mathematical ideas. Teachers must consider "comparing new mathematical ideas with others" if they intend on improving their teaching skills after meeting the basic requirements.

The relationships among "reasoning," "connection," and student engagement as perceived by elementary school students were estimated as a model by assuming that both reasoning and connection significantly predict student engagement. This model demonstrated a good fit to the data (CFI = .999; TLI = .999; RMSEA = .004); however, the path of connection to student engagement was not significant. The path was removed without dropping the model fit to achieve a parsimonious final model (Figure 3). The relationship between instructing mathematical ideas and student engagement differed depending on the factors. Engagement was strongly dependent on "reasoning;" however, engagement was not directly dependent on "connection."

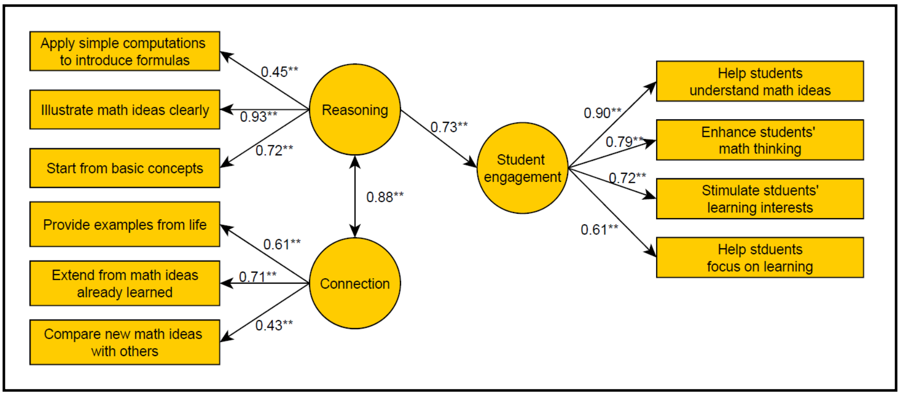

Figure 3. Relationship between instructing math ideas and student engagement

$^{**}p < .01$. $^{*}p<.05$.

Relationships between representing mathematical ideas and engagement

Table 2 displays the students' opinions regarding the representations elementary school teachers should use when delivering mathematical ideas; this table also shows the EFA results. The scale of representing mathematical ideas also comprised 2 factors. The first factor was "imagination representation," which involves the representations that require students' imagination for the students to connect to mathematical ideas. The second factor was "partial-characteristic representation"; this type of representations exhibit partial characteristics of a mathematical idea, and students approach the whole idea through these characteristics. The loadings of "things in real-life" on Factors 1 and 2 were both greater than .32, indicating that "things in real-life" reflects the connotations of both factors (Tabachnick & Fidell, 2013).

Table 2. Descriptive information and pattern coefficients of representing math ideas

Items	Pattern coefficients		Checking percentage
	Factor 1	Factor 2	
Stories	0.772		62%
Graphs	0.518		81%
Things in real-life	0.431	0.553	87%
Examples		0.828	92%
Metaphors		0.579	87%
Objects in surroundings		0.578	87%
Formulas		0.342	73%

The weighted average percentages of "imagination representation" and "partial-characteristic representation" were 74% and 87%, respectively. Furthermore, "partial-characteristic representation" obtained a higher endorsement ($p < .01$, obtained by conducting a pair-sample t test) than "imagination representation." The checking percentage of each item embedded in "partial-characteristic representation" was at least 87%, and there was only one exception. More than 90% of the students endorsed the use of "examples." "Formulas" demonstrated the lowest checking percentage (73%). According to the perceptions, a good teacher must combine various representations. This is consistent with the argument in the literature, that each representation has specific strengths and elucidates different aspects of one mathematical idea, and their combined use can be a more effective tool to support students' understanding (Kaput, 1992; NCTM, 2000). Moreover, it is surprising that the elementary school students did not endorse the representation "stories" very much.

The model for examining the relationship between representing mathematical ideas and student engagement demonstrated a good fit to the data (CFI = .978; TLI = .972; RMSEA = .002). Figure 4 illustrates the relationships among "imagination representation," "partial-characteristic representation," and student engagement. Student engagement was significantly dependent on both factors of representing mathematical ideas; however, the relevance of "imagination representation" regarding student engagement was slightly lower than that of "partial-characteristic representation."

Conclusion

Student engagement is critical to educational success in mathematics and the development of mathematical literacy. Encouraging students to engage in mathematics is a challenge for elementary school teachers. This study used a nationwide sample to examine the pattern and determinants, regarding conveying mathematical ideas, of student engagement for Taiwanese elementary school students in Grades 4 to 6. Two factors captured instructing mathematical ideas: "reasoning" and "connection;" "reasoning" involves instructing students by helping them understand the meaning of the mathematical ideas, and "connection" involves enabling students to establish connections among the ideas. Students perceived both factors as the aspects that a good teacher should focus on; furthermore, "reasoning" registered a higher endorsement than "connection." SEM showed that "reasoning" played a more crucial role than "connection" in enhancing student engagement. Representing mathematical ideas was also captured by two factors: "imagination representation"—which involves the representations that require students' imagination to enable connection to mathematical ideas—and "partial-characteristic representation"—which exhibits partial characteristics of a mathematical idea; students approach the entire idea through these characteristics. Student engagement was significantly dependent on both factors; however, "partial-characteristic representation" obtained a higher endorsement.

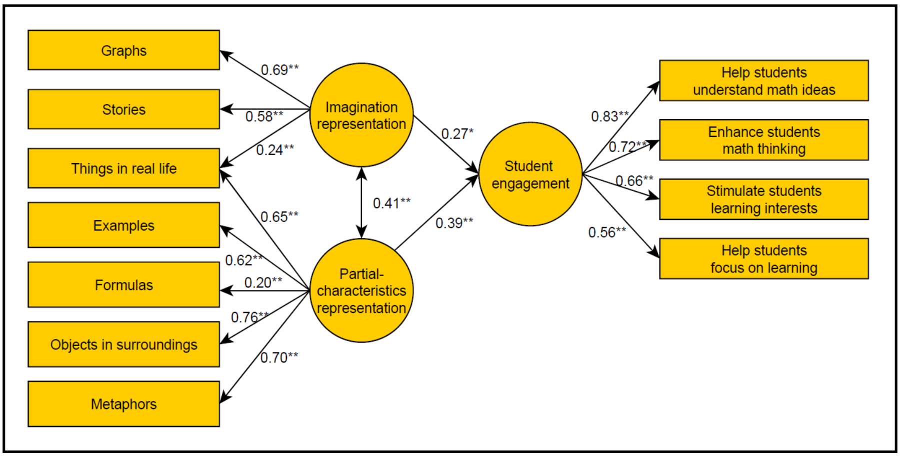

Figure 4. Relationship between representing math ideas and student engagement
$^{**}p < .01$. $^{*}p<.05$.

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