Functions and equations: Developing an integrated curriculum with the required mathematical activities

Tatsuya Mizoguchi, Tottori University, Japan

Introduction: ***Background to the problem***

In the conventional Japanese middle school mathematics curriculum, students usually learn functions after equations in all school grades. In the 7th grade, they learn about linear functions that are limited to proportionality ($y=ax$) and inverse proportionality ($y=a/x$) after learning about linear equations ($ax+b=0$). In the 8th grade, they are taught linear functions in general ($y=ax+b$) after being exposed to simultaneous equations. In the 9th grade, they learn about quadratic functions, limited to those that are proportional to the square ($y=ax^2$), after exposure to quadratic equations ($ax^2+bx+c=0$, *limited to real solutions*). On the other hand, a variety of domestic and international surveys have established that Japanese middle school students are presently unable to construct equations and subsequently represent them graphically to solve a given problem situation, even though they are able to solve the given equation and represent it graphically.

Hence, the current arrangement of the curriculum manifests students' problem solving difficulties, *i.e.*, their inability to use a graph as an effective tool when attempting to express the relation between the quantities of a problem situation algebraically in the form of an expression. In addition, students are unable to use the interaction between an equation and its graph as a standard to assess the validity of the solution of the problem. In the current curriculum, these activities are placed in the final part of Linear Functions unit[*1], which is irrational and uneconomical. Particularly, in Japanese elementary school, *a figure of tape* or *a figure of segment* (for *addition* and *subtraction*), and a *double number line* (for *multiplication* and *division*) are used as tools for deciding which operation to use (Fig.1). Therefore, in the middle school, we expect our students to be able to utilize a graph as an effective tool for building an equation, especially for their *future construction*. Further, it is also considered to be an important period for students to acquire such a capability because the middle school is the final compulsory education in Japan.

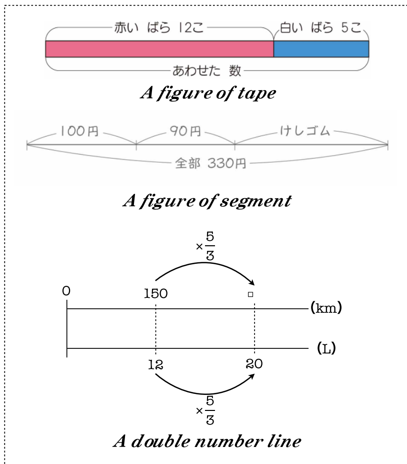


Fig. 1. Tools for deciding operation

The purpose of the study is to develop a new local curriculum, *i.e.*, an instructional unit (*Functions* and *Equations*) with the aim of presenting the mathematical contents required to teach students to use equations and graphs as effective tools for problem solving. To date, to achieve the abovementioned purpose, we have begun editing experimental textbooks, and have continued to develop lesson studies based on these edited books. We have already verified the feasibility of a new curriculum containing this new instructional unit through actual lesson studies (Yamawaki, Yamamoto, & Mizoguchi, 2013; Mizoguchi, 2014). On the basis of these studies, this paper proposes the mathematical activities that would need to be included in this instructional unit in each grade.

Methodology and theoretical frameworks

Experimental textbook

First, for each grade we produced a new integrated unit named "*Functions and Equations*" following a restructuring of the conventional units Equations and Functions that are currently being taught separately. This is intended to foster students' attitude toward and ability to effectively solve equations and draw graphs to use as a tool for problem solving. In addition, as described in the following sections of this paper, the experimental textbooks were edited to incorporate the new reorganized curriculum.

Under the School Textbook Examination Procedure in Japan, textbooks usually have the following implicit functions: to enable students to learn by way of *self-study* (especially during times of absence from school), and to guarantee *quality lessons* for teachers. Therefore, when realizing the Japanese Course of Study, teachers tend to implement their lessons (instructional plan for each unit[*2]) based on the way in which the units are organized in the textbooks.

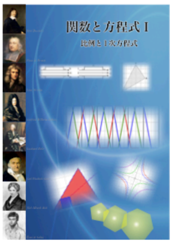
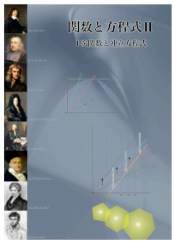
In contrast, the following is observed in the experimental textbook: a direct determination of how to use the textbook in the lesson, which involves a concrete implementation of research results (outcomes); *i.e.*, the *interaction of theory and practice*. Mizoguchi (2013*a*) revealed the methodology adopted by the experimental textbook to be that shown below:

a) An embodiment of the teacher's, or the textbook author's, thought process (therefore such a thought process is required);
b) Closely related to teaching and learning activities (the didactic situation);
c) As opposed to point (b), aspects of the mathematical content relating to knowledge beyond the individual classroom space (*i.e.*, beyond the mere contents of each lesson plan);
d) The examination and embodiment of the interaction of mathematical activities expected by the teacher (the textbook author) and performed by the learner;
e) Verification of the consistency of mathematical contents and activities in each unit, and between the different units.

It is essential to edit the experimental textbooks to demonstrate new developments in the curriculum, and such experimental textbooks may thus themselves be presented as the results of the study (*cf.* Sugiyama, *et al.*, 2003).

In this research project, the above experimental textbooks[*3] (Table1) have been edited. (*cf.* Mizoguchi, *et al*. 2012)

Table1 List of the experimental textbooks

	Functions and Equations I	*Functions and Equations II*	*Functions and Equations III*
Book Cover			
Grade	7	8	9
Published	*1st*: 2010 *revised*: 2011, 2013, 2014	*1st*: 2011 *revised*: 2013	*1st*: 2012
Conventional Unit of Ordinary Textbooks	‣ linear equations ‣ proportionality & inverse proportionality	‣ simultaneous equations ‣ linear functions	‣ quadratic equations (limited to real solutions) ‣ quadratic functions limited to the proportion to the square ($y=ax^2$)

Lesson study as an empirical method in the curriculum development

The TIMSS Curriculum Model (Mullis & Martin, 2013) is a typical framework commonly used in curriculum research. The discussion is based on this framework (Fig. 2).

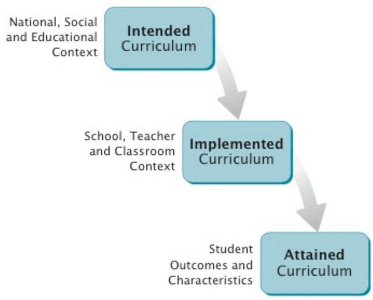


Fig. 2. TIMSS curriculum model

A diversity of levels is observed in research relating to curriculum development. In the case of a national curriculum, such as the Japanese Course of Study, even if the contents list of learning material is presented in a particular sequence, the actual implementation of such contents is not necessarily explicitly indicated. For this reason, many teachers consider a textbook to represent a manifestation of an "Intended Curriculum." The experimental textbooks that formed part of this study can be regarded as the actualization of the "Intended Curriculum."

On the other hand, in practice these experimental textbooks need to proceed beyond the "Intended Curriculum" to reflect the "Implemented Curriculum," *i.e.* the *lesson details*. However, in reality the approach followed for each lesson would differ in each classroom and simply observing "a general lesson" without adjustments, similar to the way in which a simple shape such as "*a general triangle*" would be observed, would not be possible. In this point, it can be said

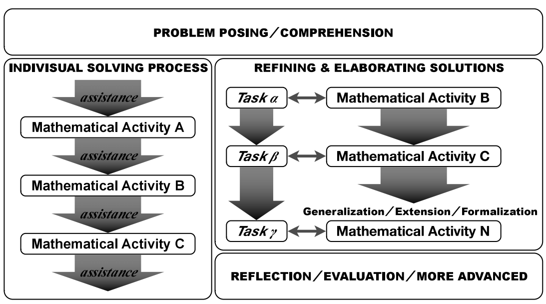


Fig. 3. Problem solving lesson model

that it is difficult to examine the "Implemented Curriculum." The experimental textbooks developed in this study are based on an innovative lesson view and lesson model (Mizoguchi, 2013*b*; Fig.3), which considers the "Intended Curriculum" to tacitly include the "Implemented Curriculum."

Even though the "Intended Curriculum" is excellent, whether it is realizable is a different matter. Implementing the lesson depends on the teacher's ability to do so. Further, "*coordination*" of the "Intended Curriculum" within the actual lesson (*i.e.*, the Implemented Curriculum) is demanded. In many curriculum development studies, they tend to overlook this point, or to insufficiently focus on the level of difficulty to allow implementation over a long period of time. This study aims to develop a curriculum empirically with emphasis on this point.

Three categories of mathematical activity

Here, I describe the notion of a *mathematical activity* in this study. In Japan, "mathematical activity" first appeared in 1998 edition of the Course of Study, and has appeared in it ever since. Moreover, in the present edition of the Course of Study, it appears at the start of the goal stated for mathematics, as a subject, in each school levels. Although they are slightly different for the different levels of description by school/grade, "mathematical activities" as seen in the Course of Study are positioned depending on the content as a whole. Although this would seem to suggest that the Japanese Course of Study has no clear "axis of process," "mathematical activity" has originally a nature as the process concept for instruction and curriculum. When mathematical activity is considered a process for the purposes of curriculum development, three categories are assumed, namely "*mathematical activity in a lesson*," "*mathematical activity through the unit/intra-unit*," and "*mathematical activity through the curriculum/inter-unit*," all of which are related reciprocally as shown in Fig.4. (*cf*. Miyakawa, *et al*., 2015)

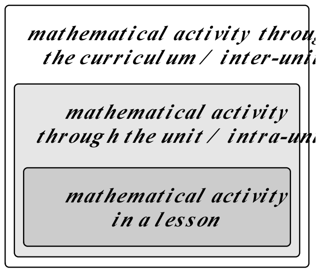


Fig. 4. Categories of mathematical activities

The concept of a "mathematical activity in a lesson" was previously defined as an expected problem solving activity in a lesson by Mizoguchi (2013*b*). However, in this study, for the purposes of curriculum development, "mathematical activity through the unit" must be considered, rather than "mathematical activity in a lesson." Nonetheless, it is evident that their validity is verified through "mathematical activities in a lesson" as a practical demand in lesson studies.

The reflection on mathematical content knowledge and a new proposal

A fundamental concept

In this study, we propose that the graph of a function be considered a tool for mathematical problem solving. In many cases, students would be able to solve their problems[*4] without much effort, if only they were able to construct (an) equation(s). However, an inability to set up the necessary equations, would lead to an inability to solve a problem. Hence, it is pointed out that students tend to experience difficulties in setting up (an) equation(s) based on the problem situation.

Prior to the 7th grade, students have learned to use tables and figures/drawings as problem solving tools. Building on these previously acquired skills, they are expected to further increase their problem solving potential by additionally learning the method of using graphs. That is, they are expected to improve their problem solving skills such that they are capable of setting up equations by using graphs or, if possible, to solve the problem by using the graph itself.

In another investigation, when 6th grade students, who had not yet been taught equations, were posed the word problem, which was an application of an equation presented to the 7th grade students, only those students who thought of changing the quantity of the problem (*thinking dynamically*) were able to solve it. This shows that students may be able to solve problems with "*functional thinking* (*grasped quantities dynamically*)" even if they are not necessarily familiar with equations or the procedure required for obtaining their solution.

On the other hand, it is difficult to expect students to solve the equation problem with functional thinking, especially by *using the graph*, in a conventional curriculum. This is because instructions are given from the standpoint that the equation problem is most skillfully solvable by that particular process. There was a tendency to categorize problems by saying that "this is an equation problem" or "this is a function problem," and to issue instructions for the solving process accordingly. As a result of this tendency, students have developed negative emotions towards comprehensive problems containing a mixture of algebra, functions, and geometry; in other words, we were confronted by our inability to sufficiently cultivate students' diverse problem solving abilities, which is not the aim of our mathematics education.

The latent issues in the conventional curriculum

For example, let us consider the following problem.

Prob.1: A train requires 60 seconds to cross a railroad bridge that is 1280 m long. In addition, it requires 90 seconds from beginning to finish for traversing a tunnel 2030 m long. Provide the length and speed (km/h) of this train. The speed of the train remains the same throughout.

The tendency is to instruct students to solve this problem by setting up the equation

$$\frac{1280+x}{60}=\frac{2030+x}{90}$$

using the segment figure (Fig.5) and the relationship

"distance ÷ time = speed"

which is the so-called "word formula." Of course, finding the solution to this problem is not problematic in itself. However, students often are unable to represent a problem situation in the form of a figure. This is a pedagogical issue in both elementary and middle school mathematics. It is anticipated that representing a problem as a figure is much more difficult in middle school because of the increased complexity of problems. In addition, although "distance ÷ time = speed" is regarded as a "*definition*" in the case of Prob.1, what is referred to as a "word formula" usually specifically applies to particular situations. (I essentially resist

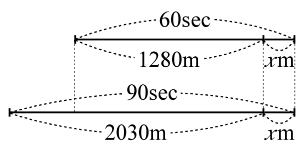


Fig. 5. Figure of segment for Problem 1

even referring to a "word formula" as a mathematical tool.) Therefore, if a student is unable to recall such a "word formula," it is anticipated that they would be unable to solve the problem. In this case, the student would never be able to solve to an unfamiliar problem, and would only be able to solve those they have experienced before.

Admittedly, the two above-mentioned issues generally present themselves in middle school mathematics. One of the reasons for this situation is the lack of progress of the problem-solving approach beyond the use of figures/drawings or "word formulas," even though the level of mathematical content continues to develop. On the other hand, learning about functions should be positioned as a tool for the examination and analysis of a phenomenon in middle school mathematics. As a result, this does not correspond to the reason for teaching students equations and functions.

Hence, as suggested in this study, we would like to educate students who are capable of putting the following approach for solving Prob.1 into practice: Because the speed of the train is constant, the distance travelled by the train is proportional to the time. Thus, the journey of this train can be represented by a proportional graph (Fig.6). Here, the slope of the graph is constant, indicating that the speed of the train is constant. From the graph, the speed of the train v(m/s) is as follows.

$$v = \frac{750}{30} = 25$$

That is, 25(m/s) = 90(km/h).

The length of the train s(m) is as follows:

$$s + 1280 = 25 \times 60\ ,\ \ s = 220(\mathrm{m}).$$

These differ from the type of equations resulting from conventional approaches. However, in this study, this approach is considered appropriate for constructing an equation using the graph of the function. Following this approach, we suggest mathematical activities that are deemed important based on our fundamental concept to be included in the unit.

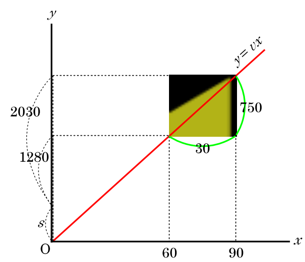


Fig. 6. Travel of the train

Mathematical activities included in the unit "*Functions and Equations*"

The 7th Grade: Functions and Equations I

In the 7th grade, a graph of a function (proportion) is introduced as a tool for setting up an equation. The main mathematical activities here are to investigate and analyze phenomena by using the graph of proportion to guide the student. Specifically, these are the following activities.

I-1) Activity to interpret the slope of the graph of proportion (*see Prob.1*)

I-2) Activity to interpret the coordinates in the proportional graph

Prob. 2: Two equally long candles (A & B) burn down uniformly in 8 hours (A) and 6 hours (B), respectively. If they are lit at the same time, when will one candle be twice as long as the other?

The problem situation is represented by the graphs in Fig. 7, the burnt part of each candle can be viewed as

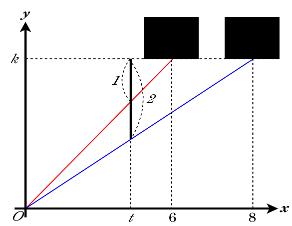


Fig. 7. Graphs of Prob. 2

the lower part of each graph (y-coordinate), whereas the remaining length after burning can be viewed as the upper part of each graph (up to k), then the following equation could be set up:

$$k-\frac{k}{8}t=2\left(k-\frac{k}{6}t\right).$$

In addition, in interpreting the graphs, it can be observed that t does not depend on k. (It means that maybe k=1, or any arbitrary value.)

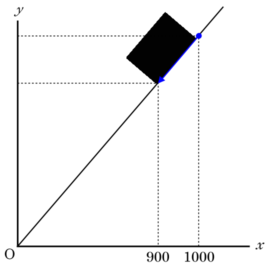


Fig. 8a. Graph of Prob. 3

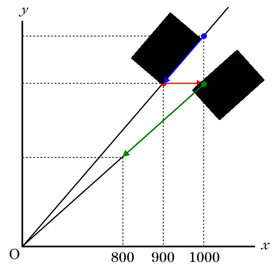


Fig. 8b. Graph of Prob. 3

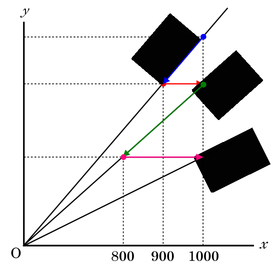


Fig. 8c. Graph of Prob. 3

I-3) Activity to interpret the graph in a synthetic way of I-1 and I-2

Prob.3: A solution of a certain density contains 1000 g of salt. First, 100 g is removed from this salt solution, and replaced with 100 g of water, and second, 200 g is then removed from this salt solution, and replaced with 200 g of water, to produce a salt solution of 8.64% weight percentage. What was the percentage of salt in the original solution?

If the original concentration of the first salt solution [A] be a%, then, because the amount of salt (y) contained in solution A is proportional to the amount of solution (x), the graph is represented as Fig.8a. Removing the 100 g from solution A does not change the concentration (slope of the graph). Next, if the concentration of salt solution [B], for which 100 g of water was added to solution A, is b%, it is clear that $a > b$, and the graph is represented as Fig.8b. Removing 200 g from solution B does not change its concentration. Furthermore, the concentration of salt solution [C], for which 200 g of water was added to solution B, is 8.64%. It is also clear that b > 8.64 (Fig.8c). Observing the y-coordinates through the behavior of the graph (especially, Fig.8c), the following equation could be set up:

$$\frac{b}{100}\times 800=\frac{8.64}{100}\times 1000,\quad b=10.8,\ \text{so}\quad \frac{a}{100}\times 900=\frac{10.8}{100}\times 1000.$$

The 8th grade: Functions and Equations II

In the 8th grade, simultaneous equations are set up as the intersection of the graphs of linear functions. Although this was the conventional approach, it is not beyond handling the development of linear functions. In our research project, by regarding linear functions as an extension of proportional relationships, we enhance the effectiveness of representing the problem situation using the graph of the function and acknowledge that this could be a problem solving tool building on the approach taught in the previous grade. Especially in the 8^{th} grade, the activity involving the construction of a graph is added to the activity requiring students to interpret a graph in the 7th grade. That is, based on the ratio of change being constant (*i.e.*, the slope does not change), the activity of translating a proportional graph in the y-axis direction, and the activity of rotating the graph around the intercept, the graph will shift to the right and either up or down if the slope is increased/decreased.

> Prob.4: Yas wants to fill a box with pears to present as a gift. The price of a pear is 200 yen for the [special] type, 150 yen for the [best] type, and 100 yen for the [medium] type. If Yas packs 36 pears of two of these types, the postage is 700 yen. With a total initial sum of 5000 yen, Yas has to have a balance amount of 300 yen or less, how many pieces of [best] and [medium] types, respectively, are packed?

This problem is considered as a function, and the different situations are represented is a graph. Let x be the number of pears, and y the price.

Buying only pears of the type [best]

$$y = 150x + 700 \;\; \ldots\ldots \; ⊖$$

Buying only pears of the type [medium]

$$y = 100x + 700 \;\; \ldots\ldots \; ⊜$$

The graph (Fig. 9*a*) is obtained from ⊖ and ⊜.

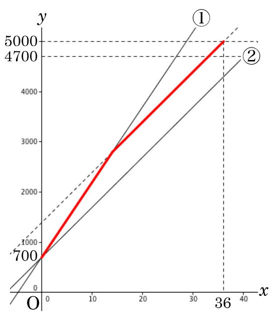


Fig. 9a. Graph of Prob. 4

If pears of the type [best] are packed first, followed by pears of the type [medium], the problem situation can be represented as a combination of two graphs, which have different slopes by translating graph ⊜ in the positive y-axis direction. We expect the students to perform the below-mentioned "graph interpreting" activities as part of their problem solving training.

<u>II-1) Activity to interpret the relation based on the increase obtained by translating the graph</u>

If the number of pears to be determined is divided into a pears of [best] and b pears of [medium] type, they will be the denoted by the increase in the x-coordinate values by each graph. In the given situation, the increases $150a$ and $100b$ on the y-axis are the prices of the pears (Fig.9*b*). The following two sets of simultaneous equations are obtained from Fig.9*b*.

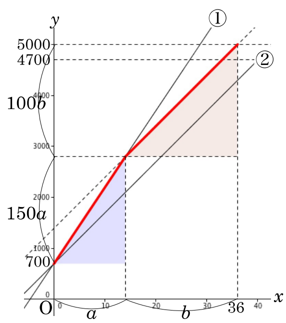


Fig. 9b. Graph of Prob. 4

$$[2] \begin{cases} a + b = 36 \\ 150a + 100b + 700 = 5000 \end{cases}$$

$$[1] \begin{cases} a + b = 36 \\ 150a + 100b + 700 = 4700 \end{cases}$$

II-2) Activity to interpret the relation based on the functional relationship obtained by translating the graph

The x-coordinate of the intersection of the two graphs represents the number of pears [best]. Here, when the price is just 5000 yen, because of the translation in the y-axis direction so as to pass through the point (36, 5000), graph ⊜ (Pear [medium]) is $y=100x+1400$ (Fig.9c). On the other hand, when the balance is just 300 yen, because it passes through the point (36, 4700), graph ⊜ is represented by $y=100x+1100$. The following two sets of simultaneous equations are obtained in this way.

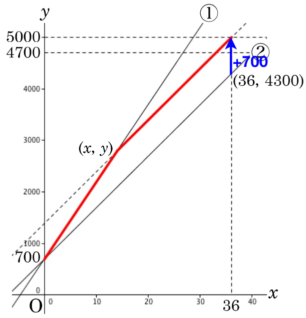


Fig. 9c. Graph of Prob. 4

$$[3]\ \begin{cases} y=150x+700 \\ y=100x+1400 \end{cases} \qquad [4]\ \begin{cases} y=150x+700 \\ y=100x+1100 \end{cases}$$

From [3] it was determined that $x=14$ and from [4] it was determined that $x=8$; thus, the combinations constitute the following seven patterns: (8, 28), (9, 27), (10, 26), (11, 25), (12, 24), (13, 23), and (14, 22). In addition, it is also possible to consider the pears of type [medium] to be packed first, followed by pears of the type [best]. In this case, it is sufficient to translate graph ⊖.

II-3) Activity to interpret the relation based on the functional relationship obtained by rotating the graph

Pears of the type [special] were also used in this problem. This condition is unnecessary for solving the initial problem. By changing the condition(s) in the problem, the type [special] was introduced with the expectation that the problem would be thought provoking. That is, it was expected that students who were able to solve the initial problem will also think "what would be the solution if the combination [medium] and [special] were used" or, "if the combination [best] and [special] were used…" by finding the position of the point of intersection of the graphs suitable for such problem situations.

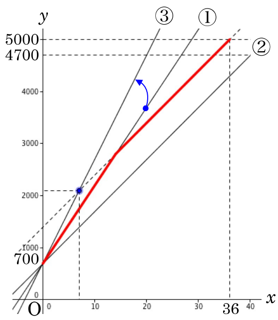


Fig. 9d. Graph of Prob. 4

If [best] in the initial problem were to be replaced by [special], what would be necessary would be to simply change the slope of graph ⊖ (Fig.9d). In graph ⊛, as compared to when combining [best] and [medium], the position of the intersection point of the graphs is closer to the y-axis. From this, it would be possible to interpret that "the number of [special] pears should be less than the number of [best] pears (the number of [medium] pears is increased)." Thus, it is possible to see without algebraic calculation that [special] is less than 8 (and [medium] is more than 28). If "change of slope" is related to simultaneous equations, the equations can easily be modified as follows.

$$[2']\ \begin{cases} a+b=36 \\ 200a+100b+700=5000 \end{cases} \qquad [1']\ \begin{cases} a+b=36 \\ 200a+100b+700=4700 \end{cases}$$

or,

$$[3'] \begin{cases} y = 200x + 700 \\ y = 100x + 1400 \end{cases} \qquad [4'] \begin{cases} y = 200x + 700 \\ y = 100x + 1100 \end{cases}$$

Solving these, the solutions that satisfy the conditions are obtained as follows:

([special], [medium]) = (4, 32), (5, 31), (6, 30), and (7, 29).

II-4) Activity to judge the relation based on the functional relationship obtained by rotating the graph

Considering that type [medium] in the initial problem changes to type [special], it would be sufficient to change the slope of graph ⊜ (Fig.9*e*). When the translated graph of [special] (⊛) passes the point (36, 5000) in the y-axis direction, as compared with the case of [best] and [medium], the x-coordinate will be larger than 36 and the y-coordinate will be larger than 5000. From this, it is interpreted that “the solution which satisfies the condition does not exist.” A similar result is obtained for the point (36, 4700). Therefore, we are able to conclude that it is not possible to buy the pears by satisfying the condition by combining [special] and [best].

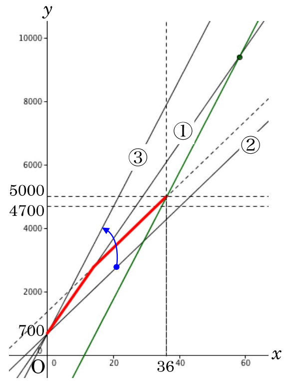


Fig. 9e. Graph of Prob. 4

The 9th grade: Functions and Equations III

Conventionally, among the quadratic functions, only the “function proportional to the square” has been taught in the 9th grade in Japan. However, this research even insists on treating the general quadratic function (*standard form*) adequately from the standpoint of developing the integrated curriculum of *Functions and Equations*. This makes it possible to link the quadratic equation and the quadratic function. However, it is important to emphasize that this research does not intend to preempt the curricula of traditional Japanese high schools, but to use it thoroughly to create a problem solving tool. Hence, central mathematical activities could provide an indication that the x-coordinate of the point of intersection of the parabola and the straight line is the solution of a quadratic equation, or it could put the range of the solution (maximum and minimum) into perspective.

Prob.5: There is a copper plate of width 20 cm. Both sides of the copper plate are bent upwards to create a rectangular-shaped gutter. What is the length in centimeters for the bent part at both ends to create a gutter with the largest area?

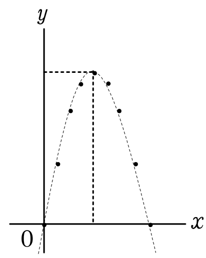


Fig. 10. Graph of Prob. 5

III-1) Activity to utilize the graph to inquire the reason of solving

Prob.5 can be solved by using a tabular method to form a perspective of the solution. However, such a solving process complicates an understanding of the solution, which is only approximated. Although it is also conceivable to draw a graph (*plot*) based on the table, a method such as this only demonstrates the process mentioned above. Then, by converting the standard form to the vertex form (completing the square), the aim should be to establish the reason for the solution to the equation when drawing the graph.

III-2) Activity to utilize the graph for generalizing the solution

Moreover, merely solving the problem does not mean the work has been completed. In mathematics education, it is important to foster an attitude of pursuing generality by changing the conditions of the solved problem. A learner would also be able to generalize given problem (and their solutions) based on prior learning. With this research, it is our intention to promote activities such as these by encouraging students to interpret and manipulate graphs. In the case of Prob.5, for example, the question would become: what would be the answer if the width of the copper plate was set to 40 cm. Further, the following two assumptions are made. First, the vertex of the parabola of the original problem and the vertex of the parabola of the modified problem occur on the same parabola. Second, the ratio of the vertical to the horizontal length when the area is a maximum becomes 1:2, a result that can also be deducted from the equation. Because the ratio of the vertical to the horizontal length is 1:2, if the vertical distance is x, then the horizontal distance is $2x$, and the area of the gutter y is $y=x\times 2x=2x^2$. That is, the equation of the trajectory (*i.e.*, *parabola*) passing through the vertex of the parabola is $y=2x^2$. (Fig.11)

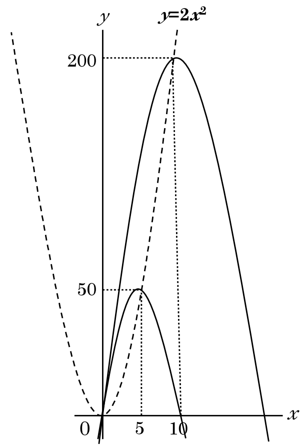


Fig.11. Trajectory of vertices

Concluding remarks

In this study, we identified the required mathematical activities that are summarized below, as part of the current stage of a long-term research project about *lesson studies*. Our fundamental idea is consistent across the three grades forming part of this study, in that a student is expected to obtain an intuitive grasp of the problem situation by initially producing a graphic representation of the phenomena with which he/she is confronted, after which he/she formulates the equation based on his/her graph.

The 7th grade students are required to perform the following mathematical activities: identifying the proportional relationship in the problem situation; recognizing the slope of the graph; recognizing the coordinates from the graph; and developing a comprehensive understanding of the graph.

The 8th grade students are required to perform additional activities to manipulate graphs (of linear functions). Mastering these activities requires the following recognition or judgment abilities; recognizing numerical relations from the viewpoint of variables; recognizing numerical relations from the viewpoint of a functional relationship (of the graphs); and judging the numerical relations by rotating the graph at a certain point.

The 9th grade students are expected to perform the following mathematical activities by learning quadratic functions (in general) and quadratic equations: using a graph to explore the reason for the solution in the expression/equation, and using a graph to generalize the solution.

Our lesson studies were conducted by taking an innovative view of the lesson and by using the lesson model as described above. We emphasize an approach for refining and elaborating upon, that is "*neriage*" in Japanese, the "*mathematical activities*" expected from teachers. In curriculum development based on lesson studies, the necessity for consistently embedding the notion of mathematical activities throughout the curriculum is recommended, with the aim of coordinating the Intended curriculum with the Implemented curriculum.

Based on this recommendation, should the standard form of the quadratic function be added to the learning content of the 9th grade as suggested by this research, it would be desirable to include "the function proportional to a square" in the 7th grade learning content. Hence, an understanding of proportional relationships forms an important prerequisite for working with the various numerical relationships, that is, the parabola should not be treated as a mere graph of a function, but as a natural development of proportional relationships.

This study has not yet concerned itself with the "Attained Curriculum," because the study is mainly interested in improving the didactics through curriculum development. In other words, it is possible to say that our central issue is to evaluate the lesson. This study emphasizes that the approach to the Attained Curriculum is also based on lesson studies.

Notes

*1: In Japan, a "unit" refers to the plan of teaching and learning according to which mathematical contents and activities are organized in relation to each subject.

*2: Although each unit is prepared and included in the textbooks, each teacher individually produces his/her instructional plan for each unit on the basis of these.

*3: All the experimental textbooks can be downloaded from the Internet free of charge: http://goo.gl/avjxop.

*4: A problem in a lesson is posed to students by a teacher as though it is their own, in other words, *devolution*.

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Tatsuya MIZOGUCHI
Tottori University, Dept of Edu, Fac of Reg Sci, 4-101, Koyamacho-minami, Tottori, Japan
mizoguci@rstu.jp