

# The teaching of mental subtraction in Matematika GASING to elementary school students

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## Introduction

The ability of doing basic mathematics is the base for understanding higher-level mathematical concepts (Kelanang and Zakaria, 2012). Basic mathematics includes addition, multiplication, subtraction and division. Failure in mastering basic mathematical concepts affects the ability of learning further higher-level mathematics.

Preliminary studies conducted by the researchers in Perumahan Bumi Elementary School, Kelapa Dua, Tangerang, Indonesia showed that students of grade V found subtraction difficult. A fifteen-minute written subtraction test was given to 13 grade V students. The mastery level achieved was 67.2%. Students' mental subtraction ability was also investigated. A mental subtraction test was given to 7 grade V students. Each student was given 8 seconds to answer each question mentally. The mastery level achieved was only 39.43%.

An innovative way of learning called Matematika GASING developed by Prof. Yohanes Surya of Surya Institute, Indonesia, tries to address the difficulties students face in understanding basic arithmetic (Surya, 2012).

Matematika GASING emphasizes students' logical ability such that they do not always have to depend on memorizing mathematical formulas. There are three stages in learning each mathematical topic with Matematika GASING: concrete, abstract and mental computation (Kusuma and Sulistiawati, 2014). At the end of each learning stage, students are required to do mental computation well, which is the desired end result in Matematika GASING. Therefore, as well as aiming to investigate how Matematika GASING helped students in learning subtraction, this research also looked into how the students' ability was in doing mental subtraction after the learning.

## Theoretical framework

### *Subtraction and mental Subtraction*

Subtraction is one of arithmetic operations in mathematics. The symbol is  $-$  (minus). It is an operation which takes a number of another number. Let  $a$  and  $b$  be numbers. Then  $a - b$  reads  $a$  minus  $b$  which means taking  $b$  of  $a$  or subtract  $b$  from  $a$ .

Mental computation is defined as a computation performed to produce exact numerical answers without using any aid such as calculating or recording device (Reys, 1985). Mental computation is often associated with mental arithmetic. Mental arithmetic is based on the mental recall of basic numerical facts and procedures which mainly counts on the memory of students, whereas mental computation concentrates on the mental processes or strategies used to obtain answers. Furthermore, mental computation includes mental arithmetic. Mental arithmetic is an important aspect that can be used in performing mental computation (Beishuizen and Anghileri, 1998). Mental subtraction in this paper means mental computation in doing subtraction.

There are various strategies developed by students in doing mental subtraction. They are categorized according to level of sophistication; counting and calculating (using or deriving from facts) strategies (Thompson, 1999).

### *Matematika GASING*

Matematika GASING is a way of learning mathematics, which is easy, fun and enjoyable. There are three important step-by-step stages to follow when learning each topic of mathematics in Matematika GASING. They are concrete, abstract and mental computation stages. The concrete stage encourages students to play and explore using concrete means. This will help students in understanding mathematical concept better (Godjali and Kusuma, 2013). This is consistent with the theories of Piaget, Bruner and Dienes (Ibrahim and Suparmi, 2012). The abstract stage is where concrete samples are then changed to abstract symbols in mathematics. Students can then read mathematical patterns and make conclusions on their own. At the end of the learning, students should be able to do mental computation well (mental computation stage). Accuracy in doing mental computation can be the result of successfully applying teacher-taught written procedures (Heirdsfield and Cooper, 2004).

Another important element in Matematika GASING is the critical point. A critical point here is defined as basic materials to be learnt and mastered on each topic taught (Surya, 2013). Students have to reach this point to be able to do mathematical questions for that particular topic with no difficulties.

### *Subtraction in Matematika GASING*

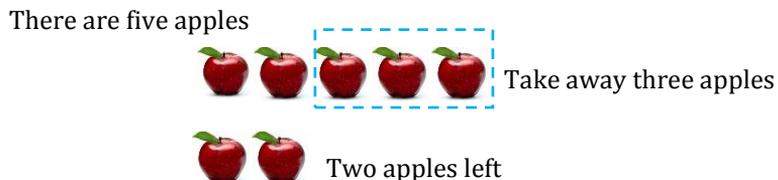
In Matematika GASING subtraction learning is divided into the following subtopics: 1) GASING critical point in subtraction, 2) subtraction of 2 digits by 1 digit, 3) subtraction of 2 digits by 2 digits and 4) subtraction of 3 digits by 3 digits.

In order to reach the critical point, there are three steps: 1) understanding subtraction concept, 2) mastering subtraction of numbers less than 10 and 3) understanding subtraction of pairs of 10. Here is an illustration of GASING critical point in subtraction.



*Figure 1. GASING critical point in subtraction*

In order to understand the subtraction concept, the learning is started by using concrete means (concrete stage in GASING). An illustration of learning  $5 - 3$  is given in figure [2].



*Figure 2. Concrete subtraction of  $5 - 3$*

This subtraction can also be described as “how many plus 3 equals 5?” This subtraction can then be modelled using fingers. Afterwards, the symbols  $5 - 3$  can be written (abstract stage). And at the end students are expected to do this subtraction mentally (mental computation stage).

### **Methods**

Methods used here was qualitative methods. Qualitative methods research is conducted under a natural condition. Data is collected directly from the source; researchers are the key instruments. It is descriptive; data collected is in the form of words or figures rather than numbers. It emphasizes more on processes rather than products or results (Bogdan and Biklen in Sugiyono, 2013).

The research subjects were 9 students of grade V at Perumahan Bumi Public Elementary School, Kelapa Dua, Tangerang, Indonesia. Instruments used were observation sheets, a written test, a mental subtraction test, a questionnaire, interviews and triangulations. The written test included 50 questions, the mental subtraction test included 25 questions (both cover every topics in subtraction). The questionnaire was given to reveal what students think of learning subtraction with Matematika GASING, in relation with levels of easy, fun and enjoyable. The questionnaire consisted of 10 yes-or-no questions.

The qualitative descriptive data analysis in this research was done by examining the data collected throughout the research. Besides tests and questionnaires, interviews were also conducted structurally to support data accuracy. They were to support the findings from questionnaire and to directly observe students’ understanding in doing subtraction. Possible difficulties students faced throughout the learning were noted; some difficulties were also uncovered through interviews and tests results. Triangulations were done to match data so that conclusions could be made.

### **Results and analysis**

#### *Learning subtraction with Matematika GASING*

Firstly, students have to reach the GASING critical point in subtraction, which is mastering subtraction of numbers less than 10, such as  $9 - 6$ . The steps to pass in order to reach this are subtraction concept, subtraction of numbers less than 10 and subtraction of pairs of 10 (for example 10’s pair of 3 is 7, 10’s pair of 1 is 9; this had been taught to students earlier in the learning of addition). The concept of number 0 is introduced here by exploration with concrete objects followed by mathematical symbols (abstract stage), so that students would understand and concluded that  $0 = 1 - 1 = 2 - 2 = \dots$  and so on.

After the critical point, the next subtopic is subtraction of 2 digits by 1 digit. There are two types, “without exchanging” (for example  $17 - 6$ ) and “with exchanging” (for example  $34 - 7$ ). Exchanging here is similar to regrouping. The subsequent subtopic is subtraction of 2 digits by 2 digits. There are also two types, without exchanging ( $34 - 13$ ) and with exchanging ( $34 - 19$ ). Last subtopic is subtraction of 3 digits by 3 digits ( $534 - 121$ ).

An illustration of  $14 - 8$  (subtraction with exchanging) is given below.

There are 1 black card and 4 white cards (concrete stage). 8 white cards are to be taken away from these cards. 8 white cards cannot be taken away from just 4 white

cards. Therefore the 1 black card needs to be changed to 10 white cards. Taking away 8 white cards from these 10 white cards leaves 2 white cards. These 2 white cards are then combined together with the 4 white cards, so there are 6 white cards now. So  $14 - 8 = 6$  (Figure [3]).

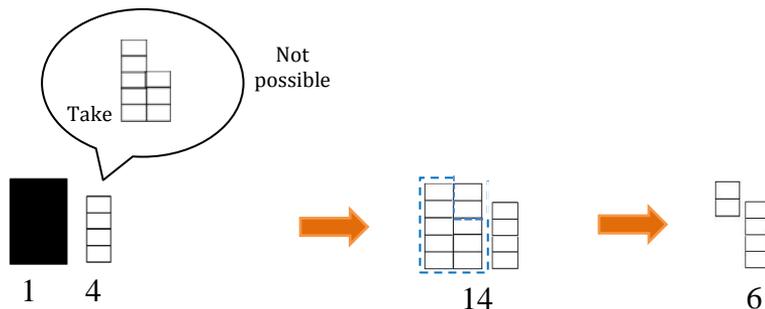


Figure 3. Concrete form of  $14 - 8$

Next, the abstract stage. First students see that it is not possible to take away 8 ones from 4 ones therefore streak 1 and write 0 and 10 (of course this is consistent with the exploration of concrete objects; 1 black card exchanged with 10 white cards). Hence, begin subtracting from the front, no tens so proceed to ones.  $10 - 8 = 2$ . But, remember there are still 4 more ones, so  $4 + 2 = 6$  (Figure [4]). So  $14 - 8 = 6$ .

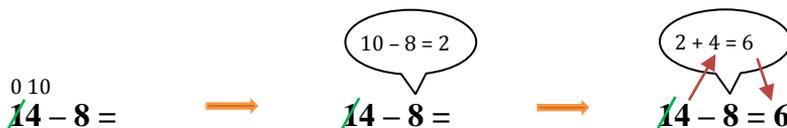


Figure 4. Abstract form of  $14 - 8$

Mental calculation process is started from the front (this allows calculations to be done relatively fast). Here, there is 1 ten. Glance to the right, to the ones,  $4 - 8$  is not possible. Therefore, 1 ten becomes 0 ( $1 - 1$ ). Next, 4 plus 10's pair of 8 (which is 2) is 6. So  $14 - 8 = 6$  (Figure [5]).



Figure 5. Mental subtraction process of  $14 - 8$

Note that DD is shorts for *dua-delapan*, 2 and 8 in Indonesia.

The process of learning subtractions of bigger numbers are similar. Blue and orange cards may also be used; each represents 100 and 1000 respectively.

#### *The Ability of Doing Mental Subtraction After the Learning*

The students' ability of doing mental subtraction and opinions after the learning was investigated through written and mental tests, questionnaires and interviews.

### The Written Ability in Subtraction

In order to see the students' written ability, a written test covering all topics in subtraction was given. The fifteen-minute test consisted of 50 questions. The ideal score was 50 (1 for each right answer and 0 for each wrong one).

*Table 1. Results of written and mental subtraction tests*

	Written Ability		Mental Subtraction Ability	
	Score	Time	Score	Time
S1	46	13'	19	08'14''
S2	41	15'	17	10'23''
S3	46	11'	22	05'35''
S4	44	15'	19	07'34''
S5	46	12'	20	09'18''
S6	46	14'	18	05'48''
S7	46	15'	18	06'58''
S8	50	15'	18	07'40''
S9	49	9'	20	03'59''
<b>Average</b>	<b>46</b>	<b>13'22''</b>	<b>19</b>	<b>07'28''</b>
<b>Mastery Level</b>	<b>92%</b>		<b>76%</b>	

Table 1 shows that the written mastery level in subtraction is 92%. This result is an improvement from the result in the preliminary studies (67.2%).

Table 1 also shows that there is one student who finishes the fastest (9 minutes) with a score of 49. On the other hand there was another student who managed to answer all questions correctly (a score of 50) but finished in 15 minutes. The average time students took to finish was 13 minutes and 22 seconds.

### The Mental Subtraction Ability

The mental subtraction ability was observed through a mental subtraction test. Students answered the 25 questions given mentally without using any aid. There were two categories observed; answering correctly and time needed in answering. Students were expected the ability to answer instantly. The data was collected by recording the test sessions using a video recorder.

The ideal score was 25 (scoring method as above). Table [1] shows that the mastery level in this case is 76% (also an improvement from preliminary study's result, 39.43%).

Based on the test and the result, it could be seen that most of students were able to answer mentally accurately. However, there were a few students who did not manage to answer instantly. These students still needed a longer-than-expected process to calculate mentally. For example, when the teacher asked the question, students would be asked to repeat the question. That question would then need to be asked several times to help students in the process of calculating mentally. The following is an excerpt of such occasion.

Teacher : 91 – 22 ?  
S7 : What did you just say?

Teacher : 91 – 22  
S7 : 91 minus 22, 91 minus 22, 91 minus 22. Seven... mmm six...  
six... six... One seven nine. Sixty nine.

It was also noted that some students had difficulties in mentally subtracting a 3-digit number by another 3-digit number with exchanging (for example  $824 - 378$ ); this requires 2 glances and 2 exchanges.

Students were not limited to a certain time when answering each question in this test. When it appeared that some students took a longer-than-necessary time to process, the question was passed and students were given the score of 0. The quickest student took 3 minutes and 59 seconds to complete the test; 20 out of 25 questions were answered accurately. The highest number of questions answered accurately was 22 (completed in 5 minutes and 35 seconds). The average time needed to complete the test was 7 minutes and 28 seconds.

#### Questionnaire on Students' Opinion about Learning with Matematika GASING

A questionnaire was distributed to seek students' opinion about learning subtraction using Matematika GASING. In particular, whether or not students find the learning easy, fun and enjoyable.

GASING stands for *Gampang, Asyik dan menyenaNGkan*. *Gampang* means no great effort or thought needed (*Kamus Besar Bahasa Indonesia Online – Great Indonesian Dictionary Online*). This is translated to easy; achieved without great effort, in English (*Oxford Dictionary Online*). *Menyenangkan* means feeling satisfied without feeling disappointed nor sad/difficult, delight, attractive (*Kamus Besar Bahasa Indonesia Online*), which can be translated as enjoyable; giving delight or pleasure (*Oxford Dictionary Online*). *Asyik* means engaged, attracted, pleased (*Kamus Besar Bahasa Indonesia Online*), which can be translated as fun; amusing, entertaining (*Oxford Dictionary Online*).

The questionnaire was developed by the researchers using indicators based on the meaning of the words *gampang*, *asyik* and *menyenangkan* (easy, fun and enjoyable). There were 10 yes-or-no questions. The score of 1 was given to a “yes” answer and 0 to a “no” answer in a positive-statement type of questions and vice versa in a negative one. Table 2 describes the results.

Table 2 shows that every students are attracted to the way the teacher taught subtraction using Matematika GASING. They did not have difficulties in following the learning, they enjoyed and were satisfied with the learning. It can be concluded that the students found learning subtraction with Matematika GASING enjoyable.

It also shows that every students (100%) are able to grasp the topic well, feel relaxed when doing the exercises and able to do them smoothly. It can be deduced that students felt subtraction could be learnt easily with Matematika GASING.

The third aspect which is fun has an average of 77.8% (100% students were keen on doing the exercises, 44.4% did more exercises than what had been assigned by the

teacher and 88.9% were willing to learn subtraction continuously with Matematika GASING).

Based on these three aspects, it can be seen that students (92.7% in average) had a positive opinion of learning subtraction with Matematika GASING.

*Table 2. Results of questionnaire*

Aspects	Indicators	No.	Answers		Percentages per Number (%)	Percentages per Aspect (%)
			Yes	No		
<i>Enjoyable</i>	1. Interested in the way teacher taught subtraction.	1	9	0	100	100
	2. Not having difficulties in following the learning.	2	0	9	100	
	3. Enjoying the way teacher taught subtraction.	3	9	0	100	
	4. Satisfied with the learning.	4	9	0	100	
<i>Easy</i>	5. Able to grasp the topic taught by teacher with no confusions.	5	9	0	100	100
	6. Able to do exercises smoothly	6	9	0	100	
	7. Feeling relaxed when doing the exercises	7	9	0	100	
<i>Fun</i>	8. Keen on doing the exercises given by teacher.	8	9	0	100	77.8
	9. Doing more exercises (more than assigned by teacher)	9	4	5	44.4	
	10. Willing to learn continuously using the method teacher taught	10	8	1	88.9	
<b>Average (%)</b>						<b>92.7</b>

#### Interviews on Matematika GASING

Interviews were conducted to support the findings from questionnaires and to recognize students' understanding in doing subtraction. According to the interviews, in general students felt attracted to learning subtraction using Matematika GASING. They liked the way the teacher taught and were satisfied with how the learning was conducted. They could do the exercises smoothly. There was one student who said it was still a bit confusing and there were three students who could not feel relaxed when doing the exercises.

During this interviews, students were asked to do 2 written subtraction questions to ensure that they understood the process of calculating subtraction with Matematika GASING. Figure 6 shows a couple of worksheets.

a. $\overset{7}{8}5 - 29 = \overset{5}{6}$	a. $85 - 29 = \dots 6$
b. $\overset{3}{3}24 - 145 = \overset{1}{1}78$	b. $\overset{3}{3}24 - 145 = \overset{1}{1}79$

Figure 6. Students' worksheets

Figure 7 shows the mental subtraction test and the learning activities.



Figure 7. Mental subtraction test and learning activities

### Discussion

The first aim of this research was to investigate how Matematika GASING helped students in learning subtraction. According to the questionnaire and interviews, students indeed found learning subtraction enjoyable, easy and fun. They found it attractive. They managed to grasp the topic well. In general, this resulted in them being keen on doing exercises.

Another purpose of this research was to examine students' ability in doing mental subtraction after learning with Matematika GASING. They were expected to be able to do mental subtraction accurately and relatively instantly after the learning. This had been reflected in their mental subtraction test results. Most students could answer mentally accurately. A few still needed a longer-than-expected process to answer mentally. Some had difficulties in mentally subtracting a 3-digit number by another 3-digit number with exchanging. However, in general it could be seen that students were able to do mental subtraction well especially in subtracting a 2-digit number by a 1-digit, a 2-digit by a 2-digit with or without exchanging and a 3-digit by a 3-digit without exchanging. Often students could even do mental subtraction of a 3-digit by a 3-digit with one exchanging. Through our observations, students needed more exercises to be able to do mental subtraction of a 3-digit by a 3-digit with exchanging.

It was mentioned that even adults found subtraction of certain minuends more difficult than others (LeFevre et al., 2006). This suggested that there are level of complexity in subtraction depending on the minuends. Further studies can be done in investigating how to achieve mastery in doing "difficult" subtraction for students; one solution may be by doing more exercises as suggested above.

This research was limited to students who had been given prior Matematika GASING learnings of addition and multiplication. The concept of subtraction is closely related to previous topics learnt, especially addition. The scope of this research is also limited to this subject. Further research can be conducted to address issues such as this in a bigger population.

### **Implication**

The outcome of this research shows that Matematika GASING can be used to train students to think mathematically relatively fast. Students were also trained to accept and comprehend mathematical concepts with clear and concrete reasons. Abstract mathematical concepts could be explained using concrete objects. Matematika GASING can be an easy-to-follow learning that students may consider attractive. Thus they would find mathematics easy, fun and enjoyable. Since students enjoyed learning subtraction using Matematika GASING, naturally other topics (division, fractions, etc.) can also be learnt using this innovative way.

Matematika GASING is one of new innovations in learning mathematics which is different to the learnings at schools in Indonesia generally. It can be used as an alternative to learn mathematics at or outside schools. It may help students or anyone who struggle with mathematics. How it may help students in learning other topics in mathematics can be investigated further.

### **Conclusion**

There are three stages (concrete, abstract and mental computation) in learning subtraction with Matematika GASING. It is important to follow the step-by-step stages. According to the results of written tests (taken at preliminary study and at the end of learning), there was an improvement in students' capability in doing subtraction. Students also had a positive opinion towards learning subtraction with Matematika GASING. Almost all thought that the learning was easy, fun and enjoyable. It can be concluded that Matematika GASING has helped students in learning subtraction.

Students were expected to answer instantly when doing the mental subtraction test. There was an improvement in students' capability in doing mental subtraction according to the results of the tests taken at preliminary study and at the end of learning. Some students had difficulties in doing mental subtraction of bigger numbers which required more than one exchanging.

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