Research on how young children construct and visualize a cube from an unfolded plane

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Background

Spatial ability has been studied from the perspectives of both psychology and mathematics education. There are two major psychological studies on spatial ability, image operation in a three-mountain task (Piaget, & Inhelder, 1967) and mental rotation (Guary, & McDaniel, 1977), both of which measure students' abilities to manipulate operations using point-of-view. The concept of mental rotation and the ability to imagine a solid body from its unfolded plane are related. Some studies indicate that mental rotation is not developed enough in young children (Shiomi, & Takeuchi, 1989), and that mental rotation in young children is influenced by physical motion (Sekiyama, 2005). Young children move their hands naturally when they make a mental rotation.

The relation between spatial ability and mathematics education has been studied in considerable detail. Spatial ability and mathematics achievement are closely related (Fennema & Sherman, 1977, 1978; Guay & McDaniel, 1977; Lean & Clements, 1981; Wheatley, 1990). Though we do not fully understand why and how, children who have a strong spatial sense perform better in mathematics. To have spatial sense, one requires spatial abilities. Spatial orientation and spatial visualization and imagery are two major types of spatial abilities (Bishop, 1980; Harris, 1981; McGee, 1979).

Spatial visualization involves understanding and performing imagined movements of 2-D and 3-D objects (Clements, 2003). Thus, young children use spatial visualization and imagery when they visualize a cube from its unfolded plane.

However, there is a lack of research studies on the development of spatial visualization and imagery in young children. Therefore, the current literature does not present a clear picture of the relationship between young children's spatial visualization skills and the resulting imagery and enriching experiences.

The purpose of this study

This study examined the spatial visualization and imagery that children utilized when they visualized a 3-D object from its 2-D unfolded plane, using a concept known as "mental rotation." The young children up to 7 years old could visualize a cube from an unfolded plane in the shape of a cross. However, other planes proved difficult (Watanabe, 2010). The study also examined what factors influence young children's mental rotation abilities, as under what circumstance they can perform that imagined movement of a 3-D object from its 2-D plane. The study further examined how young children constructed and visualized a cube from its unfolded plane.

The experience of physically making a cube is important for developing the spatial imagery skills of young children, as the enriching experience of creating the cube will expand the ability to develop 3-D spatial imagery from a 2-D plane.

First, I studied how young children construct a cube from its unfolded plane by using their hands, and defined the process of understanding the creation of a 3-D solid from a 2-D plane.

Second, I studied how these young children visualized a 3-D solid from the 2-D plane by utilizing mental rotation. I also examined the difficulties they experienced in visualizing the 3-D solid when looking only at the 2-D shape of the plane. Finally, I researched the relationship between how the children physically created a cube by hand and how they visualized a cube by using mental rotation.

Research method

Subject of research (children aged -5)

The participants were 21 children selected from a nursery in Ookubo, in Hino, Tokyo (11 boys, 10 girls), while 20 children were selected from a kindergarten located in Ochanomizu University in Tokyo (9 boys 11 girls). I examined the environmental effect on their education and subsequently, the spatial visualization and imagery skills of the young children in the study.

Teaching tools

In this study, I used a POLIDRON geometric toy for children. This toy offers ease of movement between the multiple panels, and is simple to manipulate. Therefore, it is easy for the children to assemble and convert the 2-D plane figure into a 3-D figure.

Survey methods and survey experiments

The study period was June 2014. The testing time for each child was approximately 10 minutes. I observed the children one by one, and recorded all activities. For the first, I set a cube of uniform color before the child. I also handed the child six same-colored squares that could be easily attached and removed. I instructed the child by stating, "Using six squares, please make another of the same cube that you see before you." Every child would be free to create the cube using any strategy they wished. For the second task, I set a cube comprised of three colors (with the same color squares facing each other) before the child, and handed the child six squares (2 -red, 2 -yellow, and 2 -green). I instructed the child to make an identical cube using the same colors. For the third task, I set a cube in front of the child, and showed an unfolded plane beside it. I evaluated how they answered the question, "If you were to create a shape from this unfolded plane, would it become a cube? Please think and let me know whether you think it would, or not." Each child answered, "yes," "no," or "I am not sure." I evaluated the children's response to this question when shown seven distinct unfolded planes (Fig. 1).

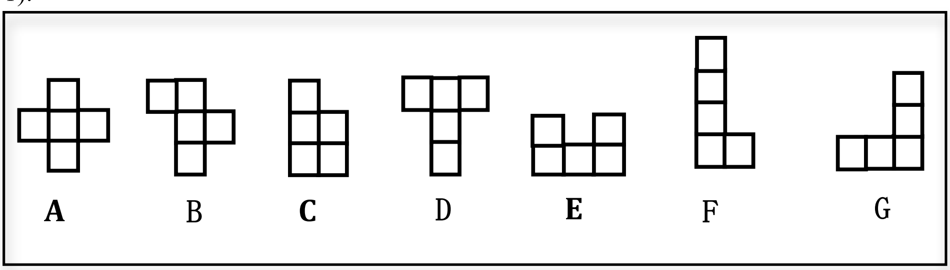

Figure 1. Seven types of unfolded planes

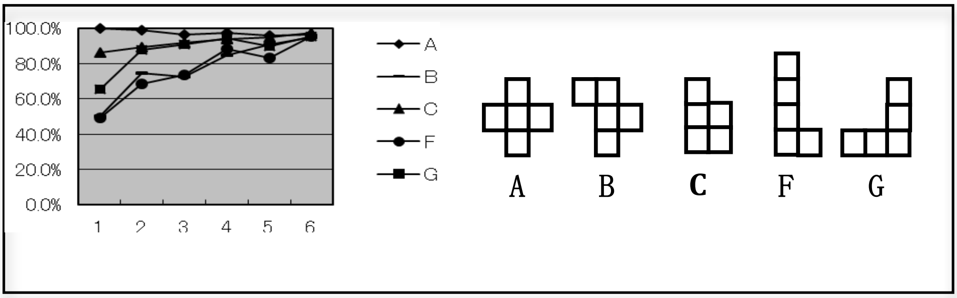

Figure 2. Percentage of correct responses using the mental rotation process with five squares

It is difficult for young children to visualize the movement of six squares, so I used unfolded planes comprising five squares. Watanabe (2010) suggests that the percentage of correct answers based on the mental rotation of five squares of unfolded planes (Fig. 2) and six squares of unfolded planes (Fig. 3) is almost equal.

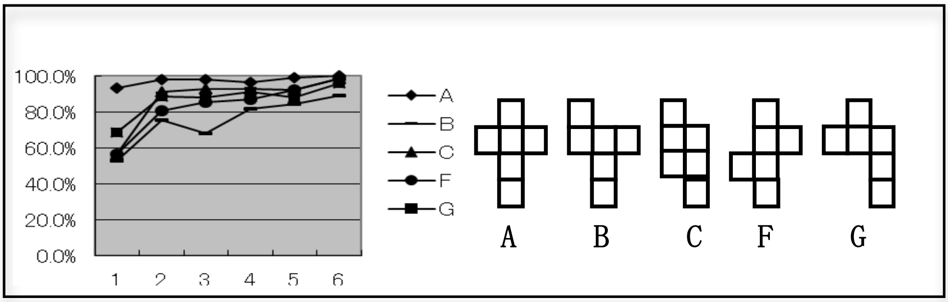

Figure 3. Percentage of correct responses using mental rotation process with six squares

Results

I observed how the children created a cube by using their hands. Five results of attempting to create the cube were recorded from my observations (Table. 1).

Table 1.Educational level and number of children who were able to create a cube

	[All same color]		[Same color face each other]	
	[Nursery]	[Kindergarten]	[Nursery]	[Kindergarten]
[Can' make]	3	0	0	2
[Using three colors]	11	9	7	4
[Same color sides set face to face]	4	4	12	7
[Using unfolded plane of four sides]	2	4	1	3
[Using unfolded plane of cross]	1	3	1	4

The first result was that a number of the children failed to construct a cube. One child attached six squares in a straight line, and attempted to make a ring (Fig. 4). This child was unable to construct a cube during the course of the observation. Another child attached six squares to create a rectangle. This child folded the rectangle along the sides in an attempt to construct a cube, but ultimately failed to do so (Fig. 5).

Figure4. Child attaching six squares straight to make a ring

Figure5. Child attaching six squares to create a rectangle

The second result was that some children attempted to create a cube using three faces attached to one apex (Fig. 6). The children could observe three faces on the sample cube. Twenty-one children made a cube of uniform color using this method. However, in the case of a cube comprising three colors, the number of children who made the cube using three faces attached to one apex decreased.

Figure6 .Child making a cube using three faces attached to one apex

In this case, one child attached the faces on the bottom, and put these faces over the sample cube to confirm that the colors were correct (Fig. 7).

The third result was the children's strategy of setting up uniformly colored sides on the bottom face to face (Fig. 8). Nineteen children used this strategy. These children could successfully create a cube with the assistance of their observation of the sample cube.

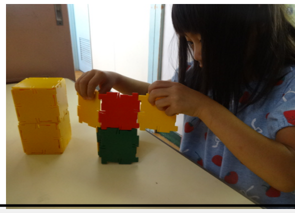

Figure 7. Child putting faces over the sample cube to confirm that the colors are correct

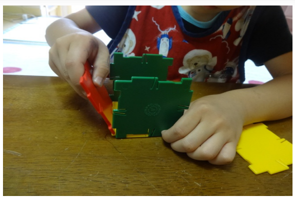

Figure 8. Child setting up same color sides on the bottom face to face

The fourth result was the children's strategy of creating a cube from an unfolded plane of four sides (Fig. 9). In this case, children attached four faces and arranged them to face each other to make sure the colors were correct.

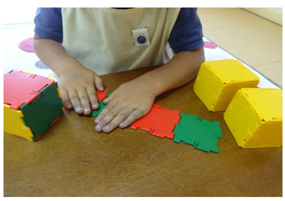

Figure 9. Child making a cube from an unfolded plane of four sides

The fifth result was the children's success in creating a cube from an unfolded plane in the shape of a cross (Fig. 10). In this case, children had previously been shown how to make a cube from an unfolded plane shaped like a cross. They already knew how to create a cube from an unfolded plane in the shape of a cross.

Figure 10. Child making a cube from an unfolded plane shaped like a cross

To assess how these young children could visualize a cube from its unfolded plane, the children were tested using seven unfolded planes (Fig. 1). The average of correct responses equaled 1.74. In the case of Plane A (Fig.1), 90% of the children provided the correct response. Therefore, it appeared that children would not be able to visualize the cube without referring to Plane A (Table 2). However, by examining the incorrect responses, I found a pattern in young children's recognition of the forms. I describe the reason below.

Plane B is correct in that it can be folded to create a cube, but 20 children respond "No." After they responded, they attempted to build a cube from this unfolded plane. However they stopped this activity after folding up one face of the edge (Fig. 11). The children did not have the experience to understand that they should move two faces together. Therefore, they could not visualize the unfolded Plane B. Also, the children appeared to be forming answers based on symmetry, for example, Planes A and D (Fig. 1) are symmetric. As a result, many children answered, "Yes." The understanding that "Plane A is correct" was observed in the children. However, Plane B (Fig. 1) is not symmetric. Therefore, the children had evaluated the answer based on whether or not the shape was symmetric.

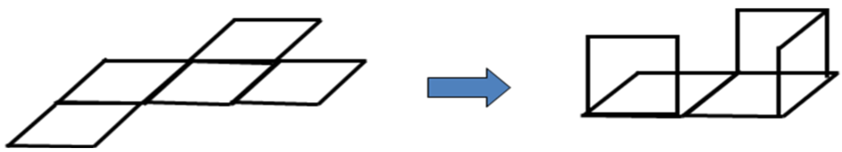

Figure 11.Child folding up one face of the edge

The response of choosing Planes C and E are incorrect, but many children answered "Yes." The children were considered to be forming answers based on the shape of a square or a rectangle as seen in the unfolded plane. The perspective of these children is similar to that in Fig. 5. Many of the children would visualize a cube based on the shape of a square or a rectangle.

Table .2.Number of answers seven questions on mental rotation

	[Yes]		[No]		[I am not sure]	
	[Nursery]	[Kindergarten]	[Nursery]	[Kindergarten]	[Nursery]	[Kindergarten]
[A]	20	19	1	1	0	0
[B]	4	6	11	9	6	5
[C]	9	7	5	2	7	11
[D]	11	5	5	7	5	8
[E]	11	7	2	5	8	8
[F]	8	3	6	1	7	16
[G]	6	3	1	3	14	14

Many of the children were observed not using a mental rotation. However, mental rotation was observed in four of the children (Fig. 12). The four children clearly had different strategies and thought processes from other children. First, they did not respond immediately. They started by thinking and moving their hands. Sekiyama (2005) reported that the ability of mental rotation in young children depends on their body movement. In this study, I observed that it is easy for young children to perform mental rotation by moving their hands. I show the results of the mental rotation responses for these four children in Table 3. The average number of correct answers without these four children was 1.7. Therefore, these four children provided significantly more correct responses than the other children.

One of the four children made a cube using the unfolded plane of four sides. Another of the children made a cube using the unfolded plane shaped like a cross. The relation between making a cube using an unfolded plane and making a cube using mental rotation was observed in two children.

Figure 12. Child visualizing the cube using a mental rotation

Discussion

I investigated the abilities of 41 children in nursery school and kindergarten in constructing a cube using their hands and visualizing a cube using mental rotation. Differences due to rearing environment were not observed. The process of children's spatial cognition can be gleaned from survey results of creating a cube using their hands and visualizing a cube using mental rotation.

Table 3 Number of correct answers of four children using mental rotation

	A	B	C	D	E	F	G	Total correct
Nursery school children A	○	×	○	○	○	○	×	5
Nursery school children B	○	×	○	○	○	×	×	4
Kindergarten children A	○	○	×	○	○	×	×	4
Kindergarten children B	○	×	○	○	○	×	×	4

Creating a 3-D shape from a 2-D shape
First, children attempted to construct a cube by connecting squares as a ring or by folding the squares. Second, children attempted to construct a cube using three faces attached to one apex. In this case, children used the 2-D view of the cube. Third, children attempted to construct a cube by setting up the sides on the bottom face to face. Children using this method knew that both sides of the surface of the bottom would come face to face. Fourth, children attempted to construct a cube from its unfolded plane of four sides. In this case, children attached four sides straight, then attached both edges and made the side of the cube. Finally, they attached the two bottoms. These children understood the two side edges would face each other when assembled.
Finally, children were able to make a cube from a 2-D unfolded plane in the shape of a cross.

These strategies are classified into two groups. One strategy was to construct the cube to be the same as the example (Fig. 6, 7, 8). The other was to make the cube from its unfolded plane (Fig. 9, 10).

Imagining a 3-D shape from 2-D shapes
The children in the study could not use mental rotation easily. They were influenced by the shape of the unfolded plane. Therefore, they believed that a symmetrical shape or a square shape would automatically become a cube. However, a few children could mentally rotate the shapes. They visualized the images for a longer time than the other children, and added a physical element by moving their hands. Hand movement assisted in the process of their mental rotation. Two of the four children who used mental rotation were successful in constructing the cube from an unfolded plane. Children are expected to be able to use mental rotation when it comes to making a cube from an unfolded plane. The enriching experiences of making cubes from 2-D unfolded planes is expected to develop the mental rotation abilities of children. In children's development, mental rotation skills are generally expected to gradually develop in spatial visualization skills and more accurate imagery.

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