Mind Mapping with ProofBlocks: Effects on student geometric level of thinking and mathematics self-efficacy

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Students' progress in geometry can be characterized using the geometric level of thinking (GLT), developed by Dina and Pierre van Hiele. These levels are:

1. *Visual*. Students determine and operate on shapes and other geometric figures based on their appearance.
2. *Analytic*. Learners recognize and characterize shapes based on their properties.
3. *Abstract*. Students establish interrelationships of properties within and among figures.
4. *Formal deduction*. The learner understands the significance of logical deduction as a method for establishing geometric truths.
5. *Rigor*. Students understand and work in a variety of axiomatic systems.

Clements and Battista (1992) suggested the existence of pre-cognition level, in which students perceive only the subset of the visual characteristics of geometric shapes and lack the capacity to form necessary visual images.

Ideally, high school students in geometry should have mastered visual and analytic levels and should be operating toward abstract and formal deduction. However, this is not always the case. According to Halat (2008) majority of high school students are still working their way to develop visual and analytic levels. This finding is consistent with the results of research on GLT of students in the Philippines (Caluya, 2000; Erfe, 1995; Liwag, 2008).

The first author considered the abovementioned studies and proposed the use of an approach, Mind Mapping with ProofBlocks (MMPB), which might help in improving student GLT. The approach also aimed to enhance student mathematics self-efficacy (MSE), a known predictor of students' performance (Nicolidau & Philippou, 2003).

Mind Mapping

Mind Mapping, developed by Tony Buzan in late 1960's, is a visual representation of concepts and their relationships. In a Mind Map, the topic is found in the center of the page with main ideas and supporting ideas radiating peripherally (Biktimirov & Nilson, 2006).

According to Brinkmann (2003), Mind Mapping has important contributions in mathematics education. The technique helps in organizing information, serves as memory aid, summarizes ideas of students, meaningfully connects new information with the given knowledge, lets the cognitive structures of students become visible, and fosters student's creativity.

ProofBlocks

ProofBlocks is a visual approach to teaching proofs, developed by Jennifer Dirksen, Nathaniel Dirksen, Jinna Hwang and Ivan Cheng in 2010. It is a set of two-dimensional blocks, which represent definitions, theorems, and postulates in geometry and can be manipulated to construct proof. Each block has a name and can be utilized if the list of labeled input/condition and output/conclusion is supplied (see Figure 1).

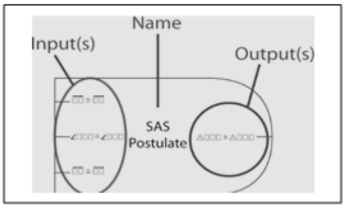

Figure 1. Two- dimensional manipulative representing a postulate. From "ProofBlocks" by N. Dirksen, 2008, http://www.proofblocks.com/ Copyright 2008 by Nathaniel Dirksen. Reprinted with permission.

These blocks can be connected if the conclusion of one block matches the condition of the other. The proof is complete when students reach a block with a conclusion that matches the statement that they are trying to prove (see Figure 2).

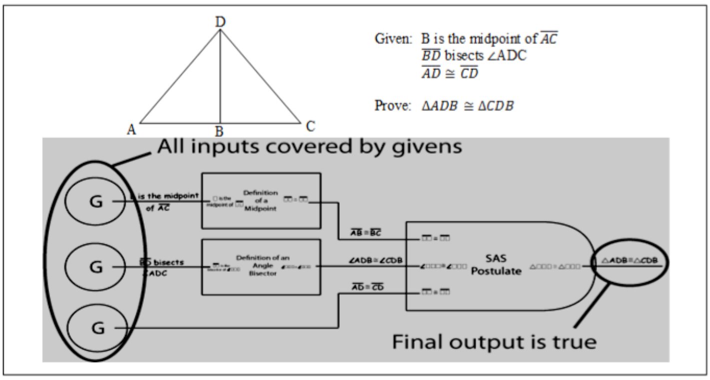

Figure 2. Sample proof using ProofBlocks. From "ProofBlocks" by N. Dirksen, 2008, http://www.proofblocks.com/ Copyright 2008 by Nathaniel Dirksen. Reprinted with permission.

Teachers use ProofBlocks as a stepping-stone to introduce a more traditional form of proof. This strategy allows students to improve their logical reasoning skills while still supported by manipulative and visual connections (Dirksen, J., Dirksen, & Cheng, 2010).

Self-efficacy in mathematics

Self-efficacy is one's judgment of his/her capabilities to achieve a goal (Bandura, 1994). In mathematics, self-efficacy is defined as a student's personal judgment of his/her mathematical abilities and is assessed by student's perceived capability to perform a given task (Bagaka, 2010; Ozgen & Bindak, 2011). According to Kabiri and Kaimanesh

(2003), MSE has a direct effect and plays an important role on mathematics achievement. Furthermore, Nicolidau and Philippou (2003) found that self-efficacy is significantly correlated and a strong predictor of student's performance in problem solving.

Visual spatial intelligence (VSI)

Student visual-spatial skill is viewed as a vital part of geometric thought (NCTM, 2000). It is the ability to perceive the visual and spatial world accurately and perform transformations to these perceptions (Gardner, 1983). Mind Mapping and ProofBlocks are known to be visual techniques in learning. According to Hoffer (1988), interconnecting visual and mathematical experiences of students makes learning more effective and enjoyable. Thus, this study sought to find if VSI moderates the effect of the teaching approach on GLT.

Purpose

The purpose of this study is to investigate the effects of Mind mapping with ProofBlocks approach on student geometric level of thinking and mathematics self-efficacy. It also examined the interaction between VSI and teaching approach in terms of GLT. This research further explored the relationship between student GLT and MSE.

Method

Sample

The study involved two heterogeneously grouped classes of third year students in different public high schools. Each section was randomly assigned as either experimental or control. A total of 74 students participated, 33 from the experimental group and 41 from the control group.

Instrument

The study used two readymade tests with permission from author/s and a researcher-made test. Cronbach Alpha was used to compute the reliability coefficient of the test constructed.

<u>Geometric thinking test (GTT)</u>

The test was made by the first author to determine the student GLT based on the van Hiele levels. Topics that were covered were points, lines, planes, angles, triangles, and parallel lines. The test was composed of 32 multiple-choice, three supply-type, and five performance-type items. Each item was categorized according to the level of thinking that it measures.

Students' scores were weighted as follows:

- 1 point for meeting criterion on level 1
- 2 points for meeting criterion on level 2
- 4 points for meeting criterion on level 3
- 8 points for meeting criterion on level 4

The student must correctly answer at least 50% of the questions for each level to be considered that he/she met the criterion. The GLT of a student was determined according to the highest level that he/she was able to pass. However, if the student failed to pass all levels prior to the highest level that he/she passed then he/she will be considered unfit for the study. Note that there was no attempt to investigate whether students operate at the fifth level, Rigor. According to Blane and Pegg (as cited in Erfe, 1995), only the most able person can perform at this level.

The highest possible score for any learner was 15 points (1 +2 +4 +8). The weighted sum scores also indicated the van Hiele level at which the fit student is operating. The test scoring procedure was made in accordance to the fixed sequence property of the van Hiele levels of thinking and was based on the works of Usiskin (1982), Erfe (1995), and Najdi (2010).

The instrument was subjected to content validation by a panel of experts from education and mathematics and was pilot tested to determine its reliability. The test was revised accordingly. Its final form had 40 items with a reliability coefficient of 0.85.

Mathematics self-efficacy scale

The instrument, created by Joaquin (2007), was used to determine student MSE. It is a 30-item rating scale distributed among various aspects of learning mathematics such as classroom instructions with the teacher, math group activities, homework, seatwork, tests, and grades. Respondents were asked to tell how true each statement was to them by encircling the number that matches their judgment – (1) not at all true of me, (2) slightly true of me, (3) moderately true of me and (4) very true of me. Six statements were negatively phrased and required a reversed way of scoring. The maximum score that a student can get is 120. Since the validity and reliability of the test have been established, with a reliability coefficient of .84, it was given directly to the students without going through pilot testing.

Spatial visualization test (SVT)

This test was created by Aptitude-Test.com–ApPro Services. It is a 15-item multiple-choice test, which asked students to manipulate 2-dimensional and 3-dimensional figures. The test was used to assess a person's VSI level. Students who scored at least 8 were considered to have a high VSI; else they have low VSI. The instrument was pilot tested with a reliability coefficient of .75.

Procedure

This was a quasi-experimental study, which utilized the pretest-posttest control group design. Two intact classes were randomly assigned to either control (conventional approach) or experimental group (MMPB approach). The first author handled the classes while teachers-in-charge observed and ensured that the first author adhered to the lesson plan. All instruments were administered to both groups at the start of the experiment and after four weeks of instruction.

Treatment

In the experimental group, a class starts with students sharing their Mind Maps from the previous lesson. Afterward, the teacher states the lesson objectives and provides activities, which will allow students to make conjectures and construct necessary observations. Subsequently, the lessons are discussed while students are expected to make their Mind Map based on the teacher's notes on the board and their understanding. Seatwork is given and proving exercises are answered using ProofBlocks. However, after three weeks, the students are transitioned to proving using two-column proofs. The class ends with students summarizing the lesson through a Mind Map.

The control group used the conventional approach. Class starts with a review by the teacher. Then, lessons are introduced in a similar manner as the experimental group. However, during discussion students are asked to copy the notes on the board using linear note taking. Thereafter, seatwork is given and proving exercises are answered using two-column proofs. The class ends with the teacher's summary of the lesson.

Results and discussion

The result of independent sample *t*-test established the initial comparability of the two groups in terms of GLT, MSE, and VSI.

Effects of instructional approach on GLT

Students' posttest on GTT was scored and analyzed. It was observed that more students from the experimental group (48%) than from the control group (24%) were able to operate at higher GLT, abstract and formal deduction (see Figure 3).

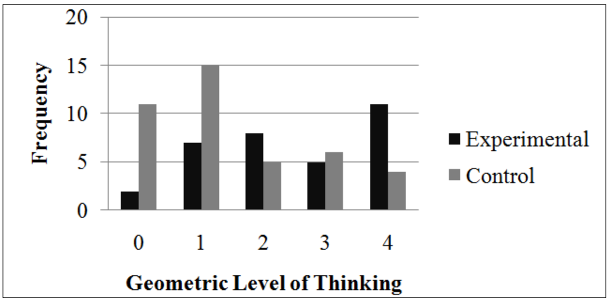

Figure 3. Frequency distribution of students at each level of geometric thinking.

The students' progress from one level of geometric thought to another was also monitored. The percentage of students who progressed from one level of geometric thinking to another was greater in the experimental group (94%) than the control group (71%) (see Table 1).

Table 1.Change in students GLT (Experimental, Control)

Pretest	Posttest				
	0 Precognition	1 Visual	2 Analytic	3 Abstract	4 Formal Deduction
0 Precognition	(2,11)	(7,14)	(8,4)	(5,3)	(8,4)
1 Visual		(0,1)	(0,1)	(0,1)	(1,0)
2 Analytic				(0,2)	(2,0)
3 Abstract					
4 Formal Deduction					

To determine whether significant differences exist in the GLT of students taught using the MMPB approach and conventional approach, the independent sample *t*-test was utilized. The mean score obtained by the experimental group (M = 7.00) was higher than the control group (M = 3.22). Results of the *t*-test showed that the difference between the scores of two groups was statistically significant, t (72) = 3.059, p = .003.

Based on the works of Usiskin (1982) and Najdi (2010), the students' score in GTT indicated their levels of thinking. Hence, those who were taught using the MMPB had significantly higher GLT than those taught using the conventional approach.

The results of the study were supported by classroom observation and interviews. Teachers-in-charge and the first author observed that when students created Mind Maps and used ProofBlocks, they were able to express their ideas, relate new topic with past lessons, organize their thoughts, and create a chain of reasoning. Moreover, the approach allowed the first author and teachers-in-charge to see and analyze how a student thinks, and provide necessary feedback. Similarly, the students noted that they were able to construct proof and develop awareness of their own knowledge through the approach. Teaching and learning processes in which students are always engaged in reflection and exposed to manipulative tasks were said to be effective in developing GLT (Caluya, 2000; Mason, 1998).

These observations were consistent with Balim, et al's. (2012) findings. They support Brinkmann's (2003) claim that Mind Mapping is an efficient tool in allowing students to experience mathematics as a system of interrelated ideas. Also, they are congruous with the study of Dirksen, et al. (2010) on the effectiveness of ProofBlocks.

A sample proof of a student transitioning from ProofBlocks to two-column proof and Mind Maps is shown in Figure 4 and Figure 5, respectively.

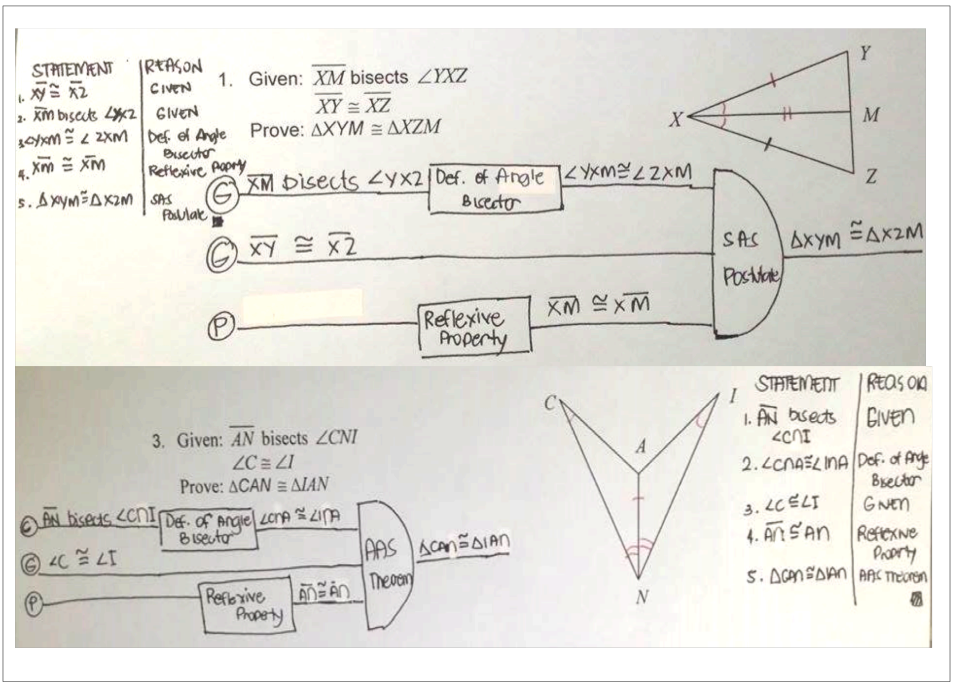

Figure 4. Sample proof of student transitioning between ProofBlocks and formal proof.

Figure 5. Mind Maps constructed by students.

Effects of instructional approach on MSE

Posttest scores of students on the Mathematics Self-efficacy Scale were subjected to independent sample *t*-test. Results revealed that the experimental group had a higher posttest mean score (M = 83.03) than the control group (M = 76.68). Results of the *t*-test showed that the difference between the mean scores of two groups was significant, t (72) = 3.169, p = .002. Hence, students who were exposed to the MMPB approach had significantly higher MSE than those taught using the conventional approach.

As previous studies indicated, a Mind Map presents a relevant concept in a clear and memorable form, and thus the students tend to get better marks (T. Buzan and Buzan, 1994). One’s success in studies is cited as one of the factors that can boost one’s personal efficacy (Bandura, 1994). The results are consistent with the studies of Budd (2003), Keles (2012), and Ramdass and Zimmerman (2008).

VSI as moderator variable

A two-way analysis of variance was applied to the posttest scores of the students on SVT and GTT to examine the interaction between the teaching approach and the level of VSI. It indicated that there was no statistically significant interaction between the two independent variables, F(1,70) = 0.002, p = .967. These indicate that VSI did not moderate the effect of the teaching approach on student GLT. Regardless of students’ VSI level, they benefitted equally from the approach.

According to Genovese (2012), skills that are necessary in Mind Mapping can be learned through constant exposure to the technique. Similarly, ProofBlocks is an easy to use manipulative (Reeh, 2011). Thus, students gain the benefits from using ProofBlocks despite their VSI level.

Geometric level of thinking and mathematics self-efficacy

The Pearson product-moment correlation was calculated to determine the relationship between GLT and MSE. The result was statistically significant, r (72) = .325, n = 74, p = .005. It indicated the presence of a moderate positive relationship between GLT and MSE. Thus, students who were operating at higher GLT were more likely to have a high MSE or vice versa.

Findings from this research are consistent with studies that examined how MSE and GLT were related to other academic variables. The study of Kabiri and Kaimanesh (2003) showed a strong and direct relationship between MSE and mathematics achievement. Likewise, according to Usiskin (1982) there is a statistically significant correlation between van Hiele levels and geometric achievement. These signify that MSE and GLT play vital roles in student's achievement.

Conclusions

With reference to the outcome of this study, the use of Mind Mapping with Proofblocks in teaching geometry significantly affects students' geometric levels of thinking and mathematics self-efficacy. In particular, the following conclusions were derived:

1. Significantly higher posttest results of students exposed to MMPB substantiate the efficiency of the approach in improving GLT and MSE.
2. The MMPB approach helps in developing the GLT of students regardless of the level of their VSI.
3. The existence of a significant and positive correlation between GLT and MSE provides evidence that it is equally important for teachers to provide as much opportunities that would enhance beliefs in one's capability and level of thinking.

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