Enhancing Van Hiele's level of geometric understanding using Geometer's Sketchpad

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Introduction

The study of geometry has been recognized as a way for students to master basic skills such as analysis, comparison, and generalization and cognitive skills in order to gain better understanding of the world (Erdogan, Akkaya, & Celebi Akkaya, 2009). The National Council of Teachers of Mathematics (2000) stressed that mathematical instruction should focus on conceptual understanding so that the students can apply the knowledge learned in various situations. In schools, however, students spend a lot of time listening to teachers' explanations without fully developing their own geometrical concepts.

Previous studies have suggested the use of technology tools in instructional processes to help students to understand geometrical concepts better and increase their motivation in learning (Chew and Idris, 2012; Chew and Lim, 2013; Dogan, 2010). Research has shown the effective use of Geometer's Sketchpad in the classroom which directly resulted in improvement of both academic achievement (Dimakos and Zaranis, 2010) and Van Hiele levels of geometrical understanding (Idris, 2009). In this study, the researchers trialed the use of Geometer's Sketchpad that utilized van Hiele phase-based instruction to enhance Year Three students' van Hiele level of geometric understanding regarding angles in a rural school in Pahang.

Research purpose

The purpose of this study was to examine whether Geometer's Sketchpad can be used to enhance the van Hiele level of geometric understanding regarding angles among primary school students. Specifically, the research was aimed at answering the following research questions:

1. Is there any significant difference in the students' van Hiele level of geometric understanding between the experimental and the control groups before the instruction using Geometer's Sketchpad?
2. Is there any significant difference in the students' van Hiele level of geometric understanding between the experimental and the control groups after the instruction using Geometer's Sketchpad?

Significance of study

This study intended to provide insight for educators to further utilize the software in the instructional process as well as using Geometer's Sketchpad in their future research to enhance cognitive aspects of the students.

Theoretical framework

Van Hiele level of geometric understanding

Van Hiele theory is a structured hierarchical process of geometrical understanding developed by Dutch mathematics educators, Pierre van Hiele and Dina van Hiele-

Geldorf. Battista (2002) suggested the van Hiele theory of geometrical understanding as the best description of students' thinking about two-dimensional shapes.

Two different numbering systems are used to name the van Hiele levels of geometric understanding in past research, namely Level 1 to Level 5 and Level 0 to Level 4 (Clements and Battista, 1992; Crowley, 1987). The researcher utilized Level 1 to Level 5 in this study so that Level 0 can be assigned to those who have not mastered Level 1 (Senk, 1989). This study involves only two van Hiele levels of geometric understanding to suit Year 3 students.

1. ***Level 1: Visual / Recognition.*** Level 1 is used to characterize the students who recognize geometrical figures visually (Crowley, 1987; Erdogan et al., 2009; Hoffer, 1983; Thompson, 2006; Van Hiele, 1986).
2. ***Level 2: Analysis.*** Students at Level 2 should be able to differentiate the geometrical figure based on their characteristics through observations and experiments (Crowley, 1987; Erdogan et al., 2009; Hoffer, 1983; Van Hiele, 1986).

Van Hiele phase-based instruction

Van Hiele proposed five sequential phases of learning in order to develop the geometrical understanding of the students, namely: information phase, direct orientation phase, explication phase, free orientation phase, and integration phase. Fuys et al. (1988) explained the phase-based instructions proposed by van Hiele and the suggested learning tasks (Table 1).

Table 1. Phase-based instructions and suggested learning tasks.

Phase-based instructions	Suggested learning tasks
Information	Student works with examples and non-examples.
Guided Orientation	Student does tasks involving different relations such as folding and measuring.
Explanation	Student is aware of the relations and tries to express them in words. Student is learning the technical language of the subject matter.
Free Orientation	Student can transfer information by knowing properties of one kind of shape and investigating these for another shape.
Integration	Student summarizes and reflects on his or her learning and actions.

Review of related literature

Chew and Lim (2013) conducted a case study of 26 Year Four pupils in Selangor, Malaysia to enhance their geometric thinking about regular polygons through phase-based instruction using The Geometer's Sketchpad (GSP) based on the van Hiele theory. A set of 20-item multiple-choice comprises the van Hiele level test that was administered to assess students' understanding about specific geometric concepts. Results of the study indicated that the students' van Hiele levels of geometric thinking about all regular polygons taught had improved significantly. The researchers further suggested the need to carry out research on primary students' geometrical thinking.

Prescott, Mitchelmore, and White (2002) conducted an exploratory study looking at Year Three students' difficulties in abstracting angle concepts from physical activities with concrete materials. Twelve teachers from five schools in Sydney participated in the study. The difficulties faced by the students were classified into four categories: matching, measuring, drawing, and describing. Abstracting the concept of 1-line angle seemed to be too difficult for Year 3 students.

Poh and Leong (2014) carried out a quasi-experimental study on 31 Year Three students in one of the rural primary schools in Pahang, Malaysia to examine the use of the Geometer's Sketchpad on students' van Hiele level of geometric thinking regarding angles. The students were selected from an intact mixed-ability class. They were then randomly assigned into experimental group and control group. Results of the study indicated that the van Hiele level of geometric thinking among students from both experimental and control groups had improved significantly. However, the difference in students' van Hiele level of geometric thinking between the two groups was not significant. Thus, this study attempts to use Geometer's Sketchpad as a tool for enhancing Year Three students' van Hiele level of geometric understanding and further guide them to grasp the concept of angles in various context.

Methodology

Research design and sample

The study was a quasi-experimental study equivalent pretest-posttest design. It was conducted in one of the rural primary schools in Pahang. All 54 Year Three students (nine years old) from two mixed-ability classrooms participated in the study but only 30 of them underwent the whole intervention process. Stratified random sampling was administered. Fifteen students were assigned as the experimental group while the remaining 15 were assigned as the control group.

Results of the independent samples *t*-tests indicated that there is no significant difference between the students in the experimental group (M = 59.80, SD = 23.11) and the students in the control group (M = 59.60, SD = 21.21) based on their mathematical achievements in the October summative tests, t (28) = 0.03, p = 0.98 at the significance level of 0.05. Research design of the study is shown as below:

Experimental group	O1	X1	O2
Control Group	O1	X2	O2

O1 represents the pretest
O2 represents the posttest
X1 represents the students learning geometrical angles using Geometer's Sketchpad
X2 represents the students learning geometrical angles using traditional method

Instrumentation

The researcher constructed a set of van Hiele Achievement Test to assess the pupils' van Hiele levels of geometric understanding before and after the intervention period, which focused specifically on visualizing the angles and identifying properties of angles.

The test comprised of ten multiple-choice items. The researcher made an assumption based on Mayberry's (1981) scoring criteria that the students achieve Level 1 in van Hiele test if and only if they can answer at least three out of five items correctly for the first five items and Level 2 in van Hiele test if and only if they score three out of five items correct for the sixth to tenth item in the test.

In the first item, students identified acute angle through visualization. In the second item, students identified both obtuse angles based on the diagram shown visually. In the third item, students observed right angles in different orientation and eliminated the one that is not a right angle. The fourth item asked students to choose a polygon with exactly one right angle. The fifth item asked students to visualize the polygon without any acute angles.

The last five items were designed to evaluate the students' understanding of the properties of angles. Students had to count the number of obtuse angles based on the exact degrees of angles in the diagram shown. Next, students were asked to state which angle is the acute angle based on the diagram given in the seventh item. The eighth item required students to imagine the turning of a vehicle in the T-junction to a specific location. In the ninth item, students were asked to predict the degree of angles between the roller coaster trails. Lastly, students had to determine a polygon that fulfills the stated characteristics.

Reliability and validity of instrument

Content validity of the instrument was cross-checked by experienced senior mathematics lecturers from the University of Malaya. They verified that the questions posed could help gather information about the understanding of geometrical angles according to van Hiele among the students. Internal consistency of the instruments was checked using Cronbach alpha test. The van Hiele Achievement Test obtained a Cronbach alpha value of .653.

Procedure

The pre van Hiele Achievement Test was administered to both the experimental and control groups before the instructional process. A brief introduction about Geometer's Sketchpad was given. Then, the students in the experimental group learned in pairs through pre-sketched Geometer's Sketchpad activities designed according to the phase-based instruction proposed by van Hiele; whereas students in the control group learnt geometrical angles through the traditional method. A similar van Hiele Achievement Test was used to assess students in both groups after the instructional period. Table 2 summarizes the research procedures.

Data Analysis

Inferential statistics of independent samples *t*-tests were used to analyze the data obtained from the pre and post van Hiele Achievement Tests using the Statistical Package for the Social Sciences (SPSS) software version 20.00.

Findings

Question 1: Is there any significant difference in the students' van Hiele level of geometric understanding between the experimental group and the control group before the instruction using Geometer's Sketchpad was given?

Table 2. Research procedures.

Groups	Research Procedures
Experimental Group	1. Pre van Hiele Achievement Test 2. Introductory lesson of the Geometer's Sketchpad 3. Phase-based instruction using the Geometer's Sketchpad Activity 1: Identify the right angles through visualization Activity 2: Identify the properties of the right angles Activity 3: Identify the acute angles through visualization Activity 4: Identify the properties of the acute angles Activity 5: Identify the obtuse angles through visualization Activity 6: Identify the properties of the obtuse angles 4. Post van Hiele Achievement Test
Control Group	1. Pre van Hiele Achievement Test 2. Instruction using the traditional method 3. Post van Hiele Achievement Test

Independent samples *t*-test (as shown in Table 3) have shown that the difference in the mean of pre van Hiele level of geometric understanding between the experimental group (M = 0.47, SD = 0.74) and the control group (M = 0.13, SD = 0.35) was not significant at the significance level of .05, where t (19.98) = 1.57, p = 0.13. This result showed that both groups have similar abilities before the intervention was administered. The effect size is 0.11, which indicated that both the groups had only a small effect on the students' achievement in pre van Hiele Achievement Test according to Cohen (1988).

Table 3. Independent samples t-test for the experimental group and control group.

Pre van Hiele Achievement Test	*M*	*SD*	SEM	df	t-values	*p*	Effect size
Experimental (n = 15)	0.47	0.74	0.19	19.98	1.57	0.13	0.11
Control (n = 15)	0.13	0.35	0.09				

Question 2: Is there any significant difference in the students' van Hiele level of geometric understanding between the experimental group and the control group after the instruction using Geometer's Sketchpad was given?

Table 4. Independent samples t-test for the experimental group and control group.

Post van Hiele Achievement Test	*M*	*SD*	SEM	df	*t*-values	*p*	Effect size
Experimental (n = 15)	0.73	0.80	0.21	28	-0.88	0.38	0.03
Control (n = 15)	1.00	0.85	0.22				

Results of the independent samples *t*-test (as shown in Table 4) indicated that there is no significant difference in the mean of post van Hiele level of geometric understanding between the experimental group (M = 0.73, SD = 0.80) and the control

group (M = 1.00, SD = 0.85), t (28) = -0.88, p = 0.38 at the significance level of .05. The effect size is 0.03. It indicated that the GPS had only a small effect on the students' achievement both the groups based on the post Van Hiele Achievement Test according to Cohen (1988).

Discussion

In this study, the results of the independent samples t-tests show no significant difference in students' van Hiele level of geometric understanding before and after the intervention period between the experimental group and the control group. However, the students in both groups had improved in terms of their van Hiele level of geometric understanding.

The students in the experimental group obtained a mean van Hiele level of 0.47 during the pretest and advanced to a mean of 0.73 during the posttest while the students in the control group obtained a mean van Hiele level of 0.13 during the pretest and advanced to a mean of 1.00 during the posttest. Based on this data, it clearly shows that the control group had improved better compared to the experimental group.

Results of this study seemed inconsistent with the results obtained by Poh and Leong (2014). This may be due to the enrichment of angle concepts such as amount of turning, intersection between two lines, slopes and so forth. Year Three students in this study seemed to not have mastered the ability to differentiate various angles in different orientation, especially when they are presented in different context.

Conclusion

In conclusion, the short duration of intervention period may been a factor that caused the results to not be significant. Students in the experimental group were still unfamiliar with the software of Geometer's Sketchpad and could not use it well as an exploration tool after a seven hour lesson. Hence, the researchers suggest that future research allow students a longer duration to familiarize themselves with the Geometer's Sketchpad.

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