Provoking students' argumentation through conjecturing in fourth-grade classroom

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Introduction

The curriculum reform recommends that teachers should provide students opportunities for conjecturing, explaining, and justifying in the classroom (NCTM, 2000). Argumentation is highly related to conjecturing, reasoning, and proving. Conjecturing launches inductive reasoning and deductive reasoning, described below in reform documents.

> A mathematician or a student who is doing mathematics often makes a conjecture by generalizing from a pattern of observations made in particular cases (inductive reasoning) and then tests the conjecture by constructing either a logical verification or a counterexample (deductive reasoning). (NCTM, 1989, p.143)

Research on reasoning, proving, and conjecturing have been increasing in recent years. The studies suggest that students should have early opportunities to incorporate conjecturing and proving into mathematical learning. However, not all tasks lead to conjecturing, and different tasks lead to different kinds of conjecturing (Cañadas et al., 2007). Students respond to mathematical tasks very differently, depending on the cognitive demands shaped by tasks designed and enacted by teachers in classrooms. This emphasis of the significance of use of tasks is not only on the task's design for conjecturing but also on the implementation of the tasks. Thus, the purpose of this study was to support teachers from third- to sixth- grade at primary school to design tasks for conjecturing in order to provide students the opportunity to engage in mathematical argumentation. The research question is to identify what argumentation looks like and how it is developed in a fourth-grade classroom.

The task design referred to the study was based on the four principles of observation, construction, transformation, and reflection (Lin, et al., 2012) and adapted from seven conjecturing processes suggested by Cañadas & Castro (2005). Cañadas & Castro (2005) propose seven stages engaged in empirical induction conjecturing: observing cases; organizing cases; searching for and predicting patterns; formulating a conjecture; validating the conjecture; generalizing the conjecture; and justifying the generalization. The four principles suggested by Lin et al. are (1) observation, (2) construction, (3) transformation, and (4) reflection. The observation-based conjecturing refers to activities that involve purposeful or systematic focus on specific cases in order to make a generalization about the cases. Construction is a principle that encourages students to construct new knowledge based on prior knowledge, which may lead to conjectures. The transformation design principle means that the task gives students the opportunities to generate conjectures by transforming given algorithms or formula. For example, after formulating the conjecture, "when you add zero to a number, you get the number you started with," students are likely to transform it to "when you subtract zero from a number, you get the number start with." The conjectures by transformation may lead

students to incorrect or meaningless statements. Thus, the reflection principle is essential to design the tasks for conjecturing.

Theoretical Framework

The theoretical framework for analyzing classrooms referred to the study relies on the symbolic interactionism (Blumer, 1969). It refers to interpretation and meaning. Interpretation of action rests on interacting with each other; individuals have to take account of what the other is doing or about to do. For example, attempts to explain one's own thinking to understand another's explanation are involved in symbolic interaction. Meaning is seen as a social product, since meaning arises in the process of interaction between people. Meanings grow out of social interaction: each individual's meanings and understandings are formed in and through the process of interpreting that interaction. Social psychologists view learning as a process that emerges during interaction (Blumer, 1969). Argumentation is seen as a social resource (Schwarz, 2009). It is considered to be an aspect of the classroom discourse that is interactively constituted by the teacher and the students. Thus, the meaning of acceptable mathematical argumentation is formed in and through the interactions of the participants in the classroom.

The relationship between learning and argumentation includes learning to argue and arguing to learn. Learning to argue involves the acquisition of general skills such as justifying, challenging, refuting, or conceding. In contrast, arguing to learn refers to achieve a specific goal through argumentation. Learning to argue about mathematical ideas is fundamental to understanding mathematics. Palincsar and Brown (1984) point out that understanding is more likely to occur when one is required to explain, elaborate, or defend one's position to others; the burden of explanation pushes one to evaluate, integrate and elaborate knowledge in new ways. Schwarz (2009) suggests that learning to argue and arguing to learn are not independent; rather, they are intertwined and often seem inseparable when they observe discussion in classrooms.

Wood (1999) defines an argument as a discursive exchange among participants for the purpose of convincing others through the use of certain modes of thought; argumentation is viewed as an interactive process of knowing how and when to participate in the exchange. The increasing studies on the analysis of students' mathematical argumentation in classrooms are adopted from Toulmin's (1958) scheme (Knipping & Reid, 2013). Kinpping and Reid's elaboration of Toulmin's argumentation scheme is useful as an analytical tool to clarify how individual students' explanations and documented learning in the classroom collectively are interactively constituted. According to Toulmin, the scheme consists of data, warrant, backing, and conclusion.

According to this scheme, the support given for a claim is the data. A warrant explains the legitimacy of the data. Backing provides further support for the warrant. The conclusion is a statement that is made as though it is certain. If a datum requires support, a new argument in which it is the conclusion can be developed. This study takes Toulmin's scheme as an approach of analyzing mathematical argumentation in classroom.

Research method

Participants and context

The task involved in the study was designed by one of the teachers who participated in the study. The task designers for the teachers engaged in conjecturing were novices, but they were mutually supported in the professional team consisting of six teachers and two researchers, the authors of the paper.

Twenty-four fourth graders in the class had learned separately the concepts of perimeter and area of a figure before engaging in the task. From the task, students were expected to learn the change of perimeter in which a small square was cut off from a rectangle. They were grouped heterogeneously in groups of 4. After being given the task, the students first worked independently and jotted down their judgment and verification on B4 paper; then they came together in groups to compare their solutions, and finally they shared their arguments with the whole class. The lesson was videotaped throughout the entire class. Each student's written work was scanned throughout the whole year.

Designing conjecturing tasks for enhancing students' argumentation

Only one of the tasks reported in this paper is composed of six subtasks described in Table 1. The task was aligned with the instructional objectives in the textbook, but it was incorporated into conjecturing. The teacher created the task, since the textbook merely provided one or two specific examples to ask students to memorize the methods of keeping the same perimeter in which a square was cut at a corner from a rectangle without exploring the reasons. The implementation of the tasks replaced the instruction of activities in the textbook, so it was not necessary to use extra hours to teach the tasks for conjecturing.

Table 1: A task for conjecturing corresponding to seven stages and four principles (Cañadas& Castro, 2005; Lin, et al., 2012)

Task: Cut a small square off from a rectangle paper and shade it.	Seven stages of conjecturing	Four principles of tasks design
(a) What do you find out after cutting a square off from the A4 paper?	Constructing individual case	
(b) Put the paper you shaded together in a group. How many ways are the papers cut in your group?	Organizing Observing	Observation Construction
(C) What did you discover? Write it down. Do you have any other ways of cutting the A4 paper such that its perimeter stays the same?	Looking for patterns Formulating conjectures	Construction. Reflection
(d) What is common way of cutting for keeping same perimeter of the new shape as the original one? How do you validate your conjectures? Show your work.	Verifying	Transformation Reflection
(e) Do your conjectures work for any cases?	Generalizing	Transformation
(f) How do you justify your conjectures?	Justifying	Reflection

Data collection and analysis

We used part of the data that was derived from a large database in a year of a project including the transcripts of a classroom episode and the copied parts of students' work. The analysis of students' mathematical argumentation was adapted from Knipping's (2008) analytical method.

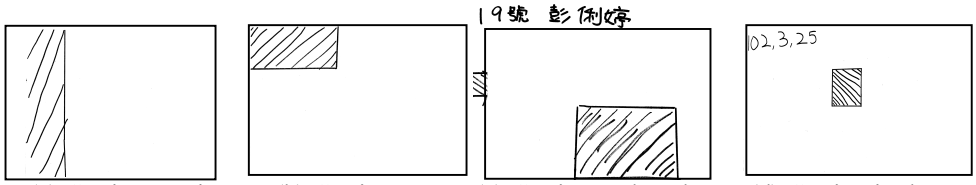
The first step was to reconstruct the sequencing and meaning of conversation. This step was to divide the classroom conversation into several segments. The arguments in each segment were identified and their sequencing was reconstructed. The second step was to analyze arguments and argumentation structures. Different functions of the arguments according to Toulmin's model (1958) such as data, warrant, backing, and conclusions were identified in this step. We used circle, diamond, and rectangle to represent data, warrant and refutation, and conclusion, respectively. The sequence of arguments with different functions formed an argumentation "stream." Parallel argumentation streams can be found to support the same conclusion. The third step was to compare local argumentations and global argumentation structures. This step was used to identify and classify the modes of argumentation based on the warrants that students employed in classrooms.

Results

The development of students' mathematical arguments was preceded by the following five stages.

Stage 1: Constructing cases as data for observing

The subtask (a) and subtask (b) were designed to provide students the opportunities of engaging in construing cases and observing the patterns. Students created and identified 4 patterns of figures by cutting a small square from an A4 paper, as shown in Figure 1. (a) If a strip was cut from A4 paper, then its perimeter became shorter; (b) If a square was cut at the corner, then its perimeter keeps the same; (c) If a square was cut at the edge of the A4 paper, then its perimeter became longer; and (d) If a square was cut in inside of the paper, then the new figure has longer perimeter.



(a) Cutting a strip off (b) Cutting at corner (c) Cutting at the edge (d) Cutting in the middle

Figure 1. Four types of figures students created

Stage 2: Formulating conjectures

The subtask (c) *"What do you discover? Write a statement according to the patterns you discovered. Show your work."* was designed to encourage each student to make individual conjectures. The problem " *Do you have any other ways of cutting such that its perimeter keeps the same?*" was designed to provide students the opportunity of engaging in finding a pattern that it keeps the same perimeter, as long as a small square

was cut at the corner of the A4 paper. In this stage, each student was encouraged to make a statement via observing, but with an element of doubt.

Students stated that the area is smaller and perimeter is smaller if a strip was cut from an A4 paper, as shown in Figure 1(a). The statement S2 was that the area is smaller and the perimeter is the same, if a smaller square was cut at the corner, as shown in Figure 1(b). The statement S3 was that the area is smaller and the perimeter is longer, if a smaller square was cut at the edge, as shown in Figure 1(c). The statement S4 was that the area is smaller and the perimeter is longer, if a smaller square was cut in inside of A4 paper, as shown in Figure 1(d).

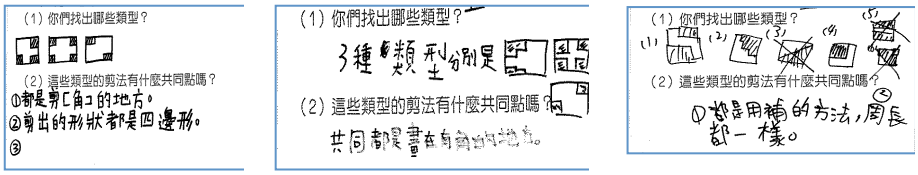
Stage 3: Examining the correctness of conjectures based on the cases
The conjectures based on observing several cases might lead students to incorrect statements. The task(c) "*What do you discover? Write it down.*" was to further examine the correctness of students' conjectures. Note that examining the correctness is different from validating the truth of conjecture that the conjecture will apply for a new case, as only the cases each individual student created.

Stage 4: Validating the truth of the conjectures
The subtask (d) "*How do you validate your conjectures? Show your work.*" gave students the opportunity to validate their conjectures to be true based on new cases. In this task, the two correct conjectures (S2 & S3) have been validated by physical operation. As figure 1(b), Nancy stated that "*if you moved the vertical line to left and moved the horizontal to up.*"

Stage 5: Generalizing the conjectures and justifying the generalized conjectures
Four statements made by students were presented in the very beginning of the parallel structures shown in Figure 3. The length of stream for cutting at the corner is longer than others. The following scenario was part of justifying the conjectures for rectangle.

25 T: Are there any other way to keep the same perimeter without cutting?
26 S14: yes, when you cut it at any corner, then it keeps the same perimeter.
27 Here [the vertical line] is the same length with this [part of the width] , and here [the horizontal] is the same length with this[part of the length].

One mode of argumentation that students used was analogical reasoning to find many ways of cutting and keep the same perimeter, as long as an irregular figure was cutting at the corner of an A4 paper, as shown in Figure 2(a), 2(b), and 2(c).



(a) (b) (c)

Figure 2. Generalizing the patterns

The argumentation streams of two statements (S_1 and S_4) disconnected to the main structure displayed in AS_1 and AS_4 in Figure 3. In AS_1, one of the students did not accept a strip as a square. Thus, the discussion was not connected to the conclusion. Likewise, in AS_4, when the teacher (T1) asked students if a small square was cut in the middle, the perimeter is longer or shorter. The answer could be either longer or shorter, depending on what the figure is. Thus, the whole discussion was stopped by the teacher. Statements S1 and S4 was not developed into conclusion. Two refutations (Ri, i=1, 2) were revealed in the argumentation structure.

In AS_2, two interventions (T_2, T_3) from teachers and a series of students' warrants (W_i, i=2,3,4,5,6,7) were argued for a square was cut at the corner of the A4 paper. They argued that the two sides of the concave figure were part of the two sides of the rectangle. Cutting a square at a corner is analogous to any other three corners. The figures can be whatever you want, as long as one cuts it at the corners. The arguments led to the conclusion.

In AS_3, only one figure resulted when a square was cut at the edge. It was not analogized by students to other three edges, since the students were not asked to give more ways to cut a square at the edge. Thus, the argumentation in AS_3 is shorter than the argumentation in AS_2.

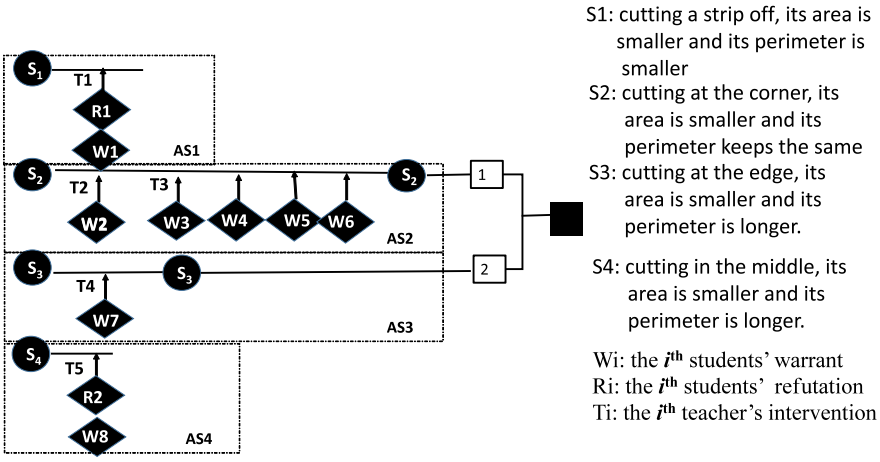


Figure 3. Argumentation structures in fourth-grade classroom

Conclusion

The task of conjecturing in the classroom contributed to three conclusions. First, the argumentation of fourth-graders was developed through five stages: constructing cases for data to identify the patterns, formulating a conjecture, examining the correctness of the conjectures, validating the truth of the conjectures, generalizing, and justifying the generalization. Second, this study contributed to the differentiation of the meaning of examination, validation, and justification involved in the argumentation. Third, conjecturing not only triggered mathematical argumentations but also enhanced students' understanding of the relationship between area and perimeter.

We found that fourth graders preferred to use empirical operation as a warrant for supporting their conjectures. Analogical reasoning was used by fourth graders in this

task to create as many figure as possible. The task involved in the study contributed to launching conjecturing because it was based on the four principles of task design for conjecturing (Lin et al., 2012). These principles shaped the task to be more open-ended while comparing the task covered in textbook. Finally, argumentation structures provided tools to better understand how students' argumentation from conjectures, supported by warrants, toward the conclusion were developed in actual classroom. The length and the complexity of argumentation structures can be served as the indicators of the degree of difficulty of arguments and the processes of argumentation.

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