Difficulty in initial stage of Filipino students' concept formation of geometric figures

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1. Introduction

For young children, one of the purposes of geometry education is to learn about space and shape. Young children's conceptions of figures have been discussed in the field of developmental psychology (e.g. Tanaka, 1966). However, in order to consider concept formation from the cognitive aspect it is said that having viewpoints from teaching and learning perspectives is important. In several previous studies in the field of psychology and education, some difficulties of children in identifying basic geometrical figures were as follows:

1. Children cannot generalize about shapes because of developmental reasons grounded in a misconception influenced by prototype example (e.g. Clements, 1998; Magara, Fushimi, 1981; Shiraisi, Ota, 1986).
2. Children cannot grasp whole figures because of influence from visual characteristics like "pointy" and "skinny" (e.g. Clements et al., 1999; Tokyo Educational Research Institution, 1959; Isobe et al., 2002).
3. There are different appearances in the same shape because of anisotropy of space. (Katsui, 1971)

In short these difficulties in figure recognition, which are caused by influences of their prototype image, visual characteristics and anisotropy of space, are related to the difficulties in generalization of figures.

What seems to be lacking is an analysis of students' actual condition as caused by these difficulties. If their condition of understanding is educational in nature particularly, developmental, we could find effective ways of teaching geometry. This study was designed to investigate young children's conception of geometric figures under this assumption. The goal was to answer two questions: (1) what kinds of difficulties in each grade do students have? (2) How do students use evidence of these difficulties?

2. Theory and research framework

2-1 Concept formation of geometric figure and generalization

How do young children build their conception of geometric figures in daily life?

Skemp (1973) states that a concept is an abstraction, something learnt that enables us to classify things. It is the defining property of a class. For example when we see a soccer ball as a concrete object, we abstract shape from some attributes like color and materials, then the shapes are categorized by focusing on commonalities and differences. After that the shapes as a class are generalized.

In this study, generalization is defined as the process of forming classes that focus on commonality among objects, and conception is defined by what students produce as a result of generalization. Vygotsky (1934) claimed that the formation of a concept entails different preconception stages. A characteristic of one stage, "Complex," is that ideas are linked in the child's mind by association or based on intuitive link based on

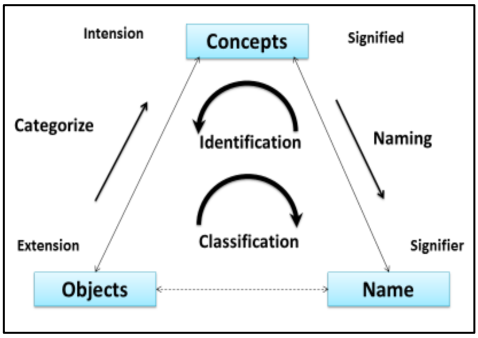


Figure 1. Function of naming (Kawasaki, 2010)

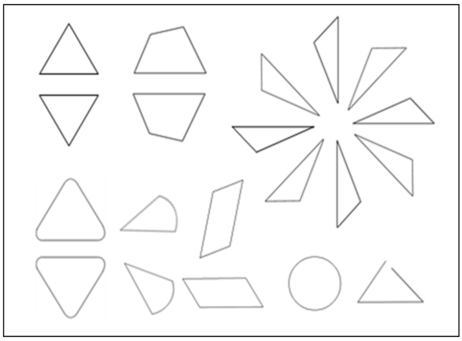


Figure 2. Shapes in questionnaire

essential attributes. Under the situation of the insufficient generalization, there are many clues to find commonality. In consequence we need to focus on students' viewpoints of their judging situation in order to describe the kinds of clues or evidence they use and how they use these.

2-2 The perspective to figure out difficulty of generalization

Figure 1 describes the function of naming. In this study, Figure 1 is used to understand the structure of concept formation. In working with triangles, students categorize shapes through similarities and differences. After that they obtain the conception of a triangle - a closed shape made of three straight lines. The name "triangle" is then given. This flow is called classification. After knowing the name, then see a new type of triangle, they judge if it is a triangle or not. This flow is called identification. Through both flows, students' conceptions of triangles become enriched. The generalization of triangles could be seen as an extension of the concept as shown on the lower left side of Figure 1.

Based on the above discussion, three viewpoints are raised, intension and extension, categorization focusing on common property and naming of shapes. Using Figure 1's interpretation, first we will focus on expanding the extension, which is one aspect of conception. Second, we describe the situation where essential attributes and nonessential attributes are mixed.

2-3 Contents and method of questionnaire

There are twenty shapes in the questionnaire (Figure 2). Each shape has some survey items, components, orientation and position (Table 1). These three survey items are connected to prototype, visual characteristics and anisotropy of space. In addition, these components are included because they are mathematical viewpoints given in schools.

Regarding the method of implementation, at first participants are asked "Is this shape a triangle?" and then make judgments whether it is a triangle or not. This situation corresponds to the flow from "name" to "objects" through "concepts" in Figure 2. The participants might use their own conception, which is could be a linguistic representation or imagistic representation and make a judgment.

Table 1. Survey items

Items		Shapes
Components	Number of sides	Quadrilateral, Parallelogram
	Condition of sides and vertices	Sector, Rounded triangle, Unclosed triangle
Orientation		Scalene triangle Eight directions: 0°45°90°135°180°225°270°315°
Position		Equilateral triangle (prototype) and upside downed equilateral triangle Sector and unstable sector Rounded triangle and unstable rounded triangle Quadrilateral and unstable quadrilateral Parallelogram and unstable parallelogram

3. Method

3-1 Participants and questionnaire

The survey in questionnaire form was conducted at the end of January in 2013. The participants were 40 first-grade students, 41 second-grade students, 40 third-grade students and 41 fourth-grade students who were selected randomly from two schools in Quezon City, Philippines.

In the questionnaire, there are five shapes per page. The total number of pages is four. The researcher asks the students "Is this shape a triangle?" in English. Then the students select the answer from three choices, "Yes," "Maybe Yes," and "No".

To understand the meaning of "Maybe Yes," the participants had time to practice judging "Maybe Yes" using "Siguro" its translation in Tagalog.

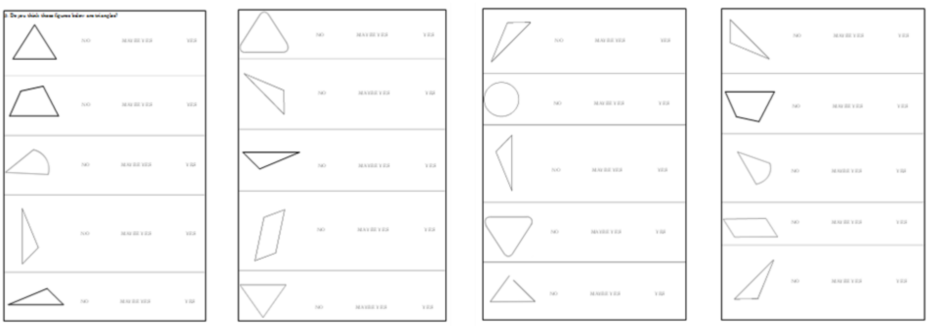


Figure 3. Questionnaire

4. Results and consideration

4-1 Extension of triangles

The mean score and standard deviation for the correct responses of each grade are summarized in Table 2. There were significant differences among grades ($F=18.1$, $p < 0.1$).

Table 2. Mean and standard deviation

	Grade 1 (n=40)	Grade 2 (n=41)	Grade 3 (n=40)	Grade 4 (n=41)
Mean	12.5	14.1	14.9	15.5
S.D	2.21	2.08	1.84	1.31

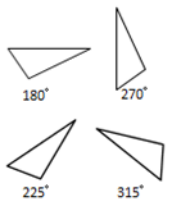
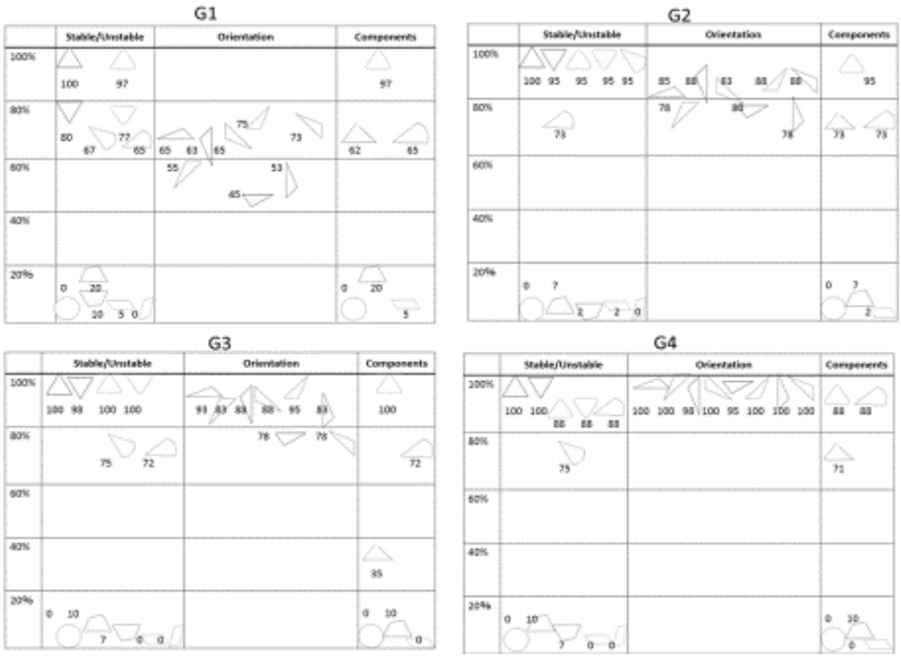


Figure 4

Figure 5 summarizes the aspects of extension in each grade. It indicates that circles and quadrilaterals including unstable ones were identified correctly. However, most of the students categorized a sector, rounded triangle and unclosed triangle as triangles. The grade three students showed the fewest misconception with only 35% of students judging these shapes as triangles. Regarding orientations, the first-grade students had difficulty identifying scalene triangles. Especially in the case of the longest side setting vertical and horizontal (*Figure 4*), the percentage of correctness observed was 45%, (180°) and 53% (270°). On the other hand, in the case of sharp angle setting above (*Figure 4*), the percentage of correctness is higher than others, 75% (225°) and 73% (315°).



Note: Percentage indicated the ratio, what percentage of students recognizes the shape as a triangle.

Figure 5. Aspect of extension

4-2 Focusing on ambiguity.

In this section, only statistically significant different points are discussed based on the analysis of answers that included three choices.

4-2-1 <u>Characteristics of the grades related to prototype</u>

1 2 3 4

Figure 6. Shapes related to prototype
(1) equilateral triangle; (2) Upside down equilateral triangle; (3) Rounded triangle; (4) Upside down rounded triangle

In Table 3, there are statistical differences among grades in the case of upside down equilateral triangle ($\chi^2(6)=21.22$, $p < .01$). According to residual analysis, grade 1 students show the highest percentage (20%) of students who say that upside down equilateral triangles are "Not triangle." Six out of these eight students think a rounded triangle is a triangle and five out of eight do not think an upside down rounded triangle is a triangle. From these results, the students who made wrong decisions in the case of an upside down equilateral triangle tended to be influenced by the horizontal base even in the case of rounded triangles.

Table 3. In the case of upside down equilateral triangle

	G1(n=40)	G2(n=41)	G3(n=40)	G4(n=41)
Not triangle	20% (8)	5% (2)	2.5% (1)	0% (0)
Maybe triangle	15% (6)	22% (9)	15% (6)	5% (2)
Triangle	65% (26)	73% (30)	82.5% (33)	95% (39)

In Table 4, the focus is on the students who judged a rounded triangle to be a triangle. The table shows that their wrong judgment gets stronger as their grade level increases (G1:61%, G2:68%, G3:73%, G4:88%).

Table 4. In the case of upside down rounded triangle

	G1(n=31)	G2(n=25)	G3(n=30)	G4(n=25)
Not triangle	23% (7)	4% (1)	7% (2)	0% (0)
Maybe triangle	16% (5)	28% (7)	20% (6)	12% (3)
Triangle	61% (19)	68% (17)	73% (22)	88% (22)

4-2-2 <u>Characteristics of the grades related to visual characteristics</u>

1 2 3 4

Figure 7. Shapes related to visual characteristics
(1) Sector; (2) Unstable Sector; (3) Scalene triangle 0°; (4) Scalene triangle 315°

Table 5 shows that there are statistical differences among grades in the case of unstable sector ($\chi^2(6)=21.65$, $p < .01$). According to residual analysis, Grade 2 students show the highest percentage (68%) of students who say that an unstable sector is "Maybe triangle." These twenty-eight Grade 2 (68%) students were excluded.

Table 5. In the case of unstable sector

	G1(n=40)	G2(n=41)	G3(n=40)	G4(n=41)
Not triangle	32.5% (13)	5% (2)	25% (10)	7.3% (3)
Maybe triangle	30% (12)	68% (28)	52.5% (21)	51.2% (21)
triangle	37.5% (15)	27% (11)	22.5%(9)	41.5% (17)

The visual features of unstable triangles are "Pointy" and "Curved line" (Figure 7). In order to figure out the students' viewpoints in identification, the results are compared between sectors 0° and 315° (Table 6). It is clear that more than half of the students say these shapes are "Maybe triangles" (52% for Sector, 52% for 0°, 57% for 315°). Thus these students have a tendency to judge uncertainly. However, there are some students who judge correctly (29% for Sector, 29% for 0°, 29% for 315°) and some students who select wrong answers (19% for Sector, 19% for 0°, 14% for 315°). These results indicate that it is easier for these students to make judgments in the case of sector, 0° and 315° because of a simple visual feature, either "Pointy" or "curved line."

Table 6. In the case of Sector, 0°, 315° in Grade 2

	Sector (n=28)	0°(n=28)	315°(n=28)
Not triangle	29% (8)	19% (5)	14% (4)
Maybe triangle	52% (15)	52% (15)	57% (16)
triangle	19% (5)	29% (8)	29% (8)

4-2-3 Characteristics of the grades related to orientation

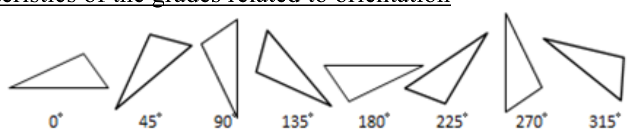


Figure 8. Shapes related to orientation

There are statistical differences among grades in the case of "Not triangle" in any direction in grade 1, "Triangle" in any direction in grade 4, "Maybe triangle" in 315°in grade 2, and "Maybe triangle" in 0° in grade 3. Figure 9 shows the conviction degree change among grades. First-grade students have difficulties with some directions. Both percentage of correctness and conviction degree are lower than other grades. In grades 2 and 3, the percentages of correctness in any direction was higher than in grade 1, but the ratio of "Maybe triangle" is still high, therefore the conviction degree is low. In grade 4, both percentage of correctness and conviction degree rapidly becomes higher than in other grades. This shows a gradual increase of students' understanding of triangles as they progress through each grade.

5. Conclusion

Let us summarize the actual situation of difficulties focusing on viewpoints students had as clues in identifying triangles.

Firstly, regarding prototype, by comparing a rounded triangle and an equilateral triangle, which have items prototype image and condition of vertices, it was found that:

- ✓ First-grade students are influenced by prototype image and horizontal based triangles and not by vertices.
- ✓ In other grades, although the impact from prototypes was less than in grade 1, they still think rounded triangle-like shapes and upside down rounded triangle-like shapes are triangles.
- ✓ Judging from conviction degree, the students in grades 2, 3 and 4 who keep the image of prototype regarding a horizontal base do not see features from a mathematical viewpoint. As a result we observe the continuity of wrong judgments with strong conviction.

Figure 9, In the case of scalene triangle in orientation

Secondly, regarding visual characteristics, by comparing unstable sector and sector, scalene triangles, which have items pointy and curved line, the findings are as follows:

- ✓ Unstable sectors that have two items, pointy and curved line, is associated with more ambiguous judgments than with sector and scalene triangles.
- ✓ Students could not utilize such characteristics in the case of sector and scalene triangles properly. They make right decisions and wrong decisions.

Thirdly, regarding orientation, by comparing directions and grades it is found that:

- ✓ The scalene triangle has two factors influencing students' judgment, the position of acute angle and longest side.
- ✓ First-grade students make judgment influenced by two factors. Uncertainty in grade 1 is less than in grades 2 and 3.
- ✓ Second and third-grade students have a tendency to judge with uncertainty whereas fourth-grade students identify triangles with conviction.

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