Lesson study for improving quality of mathematics education

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Introduction

Quality Mathematics Education (QME) may align with the three levels of mathematics education programs as defined by Husen (1967): *Intended, Implemented, and Attained Curriculum*. From an international perspective, results from PISA or TIMSS are representative of the *Attained Curriculum*, while the newly implemented Common Core State Standards (CCSS) may represent the *Intended Curriculum*. However, among Husen's three stratums, the *Implemented Curriculum* seems to be the most critical in relation to QME. The TIMSS 1995 Video Study was the first attempt to focus on this stratum, examining a snapshot of mathematics classrooms from an international perspective. Although most of the book focuses on findings from the aforementioned video study, *The Teaching Gap* (Stigler & Hiebert, 1999) also includes detailed information on Lesson Study, or *kenkyujugyo* (Yoshida, 1999; Fernandez & Yoshida, 2004). The authors characterize Lesson Study as a cultural activity in Japan and as a powerful tool for teacher professional development. Since then many mathematics teachers and teacher educators have been trying to implement Lesson Study in their own countries. Numerous books and research papers have been written on various aspects of Lesson Study and the typical lesson flow of a *structured problem-solving lesson* (Stigler & Hiebert, 1999) typical of Japanese mathematics pedagogy. While considering the QME in Japan, the focus of this paper is to highlight the importance of unifying Lesson Study with *the structured problem-solving lesson*.

QME depends on quality of teaching

Three Levels of Teaching

As a way to uncover the differences in teacher expertise, Sugiyama (2008) identifies three levels of teaching—Level 1, Level 2, and Level 3:

Level 1: *Explanation*. The teachers can show important basic ideas of mathematics such as facts, concepts, and procedures.

Level 2: *Explanation and Rationalization*. The teachers can explain meanings and reasons for important basic ideas of mathematics in order for students to understand them.

Level 3: *Student-Centered Exploration.* The teachers can provide students opportunities to discover and understand basic ideas, and support their learning in order for students to become independent learners.

In order to distinguish a Level 1 teacher from a Level 2 teacher, Sugiyama (2008) provides an example of a lesson focused on dividing fractions. Similarly, Skemp (1976) uses the same example to distinguish "instrumental understanding" which is equivalent to a Level 1 teacher and "relational understanding" which is equivalent to a Level 2 teacher. On the surface, Level 3 may seem the most simple since students seem to be doing most of the work. However, in order to reach such a level, teachers must possess an underlying pre-requisite set of knowledge and competency. For example, in order to guide students in problem solving, teachers need to be aware of each student's background knowledge so that the students can tackle the particular task fruitfully. This way, the teacher can determine where to and how to begin the lesson focusing on self-discovery on the part of the students.

Building off of Shulman's (1986) pedagogical content knowledge (PCK) in order to become a Level 3 teacher, three additional categories of that knowledge base exist. The first is the knowledge of organizing lessons to adhere to the problem solving approach. The other is the knowledge of Lesson Study as a system. Finally, the third category is the recognition of the value and methodology of *kyozai-kenkyu*. This term in Japanese is to research or study teaching materials from a mathematical and an educational point of view as well as keeping the students' point of view in mind (Watanabe, Takahashi & Yoshida, 2008). Careful attention must be given to typical conceptions and misconceptions students may have because these surely affect student learning in the problem-solving process. Therefore, from a Japanese cultural perspective, in order to improve QME *the structured problem-solving lesson* through Lesson Study based on *kyozai-kenkyu* should be implemented.

Level 3 teaching crystallizes in *the Structured Problem-Solving Lesson*

There are several identifiable patterns in Japanese mathematics lessons. Becker et al. (1990) has identified eight components of a typical mathematics lesson in Japan. A similar pattern of five components has also been identified by Stigler and Hiebert (1999). It is critical to note that these patterns have been identified from an observer's point of view. Japanese teachers may not consciously follow these patterns entirely. For instance, Stigler and Hiebert's (1999) one component, *reviewing the previous lesson* is not always a necessity or an important activity from a Japanese teachers' point of view. The following activities are most often found in a typical Japanese mathematics lesson:

1. Present a problem for the day (students try to understand the problem)
2. Problem solving by the students (students try to solve the problem)
3. Compare and discuss (*Neriage* in Japanese)
4. Sum up by the teacher (*Matome* in Japanese)

These four activities or components are a kind of framework of the lesson. What the teacher actually does in each framework are as follows:

Presenting a problem for the day

The Japanese *structured problem-solving lesson* is not "Show and Practice" (Lappan & Phillips, 2009). Some may interpret "presenting a problem" to be a detailed explanation of a set of procedures used to solve the presented problem—or a demonstration on how to solve the task. In fact, in *The Teaching Gap*, Stigler and Hiebert (1999) state that, "the (Japanese) teacher presents a problem to the students without first demonstrating how to solve the problem" (p. 77). The authors continue, "We realized that U.S. teachers almost never do this, and how we saw that a feature we hardly notice before is perhaps one of the most important features of U.S. lessons — that the teacher almost always demonstrates a procedure for solving problems before assigning them to students" (p.77).

In Japan, "presenting a problem" means to make students understand the context embedded in the task as well as the corresponding mathematical conditions that would be used to solve the task. Therefore, the following principles are necessary for an ideal task:

a. It is appropriate and mathematically valuable in terms of the goals of the lesson;
b. It interests the students;
c. It is at the appropriate level of difficulty;
d. It can be solved in several ways;
e. It has a potential to elicit valuable basic wisdom (Fujii, in press).

Usually, only one task is given in a lesson. This task is carefully chosen and presented in terms of the aims of the lesson. Similarly, a conscious effort is made in choosing the numbers used in the problem. For example, when studying subtraction, specifically when a single-digit number is subtracted from a two-digit number and requires borrowing, the first problem in a first grade textbook is 13 - 9 or 12 - 9. These numbers are considered as quasi-variables (Fujii & Stephens, 2008; Fujii, in press). A quasi-variable is a number deliberately used in a general way so that it serves as a representative of many numbers, just as a variable would. Numbers are often chosen, then, based on their quasi-variable power, or how well they demonstrate a general truth. Such close attention to the specific numbers does not mean that teachers are sticking to a concrete level of thinking and encouraging students to think about things concretely. On the contrary, teachers consider the general aspect of the number—its quasi-variable aspects.

Problem-solving by the students

While students work on solving the problem independently, the teacher walks around the classroom checking and monitoring the students' work, *kikan-jyunshi*. In Japanese, this term means purposeful scanning or monitoring. Almost all teachers in other countries behave similarly—what is consciously being done may differ from one country to another. Hino (2003) identified results from the Learners Perspective Study in order to distinguish the initial efforts made by an Australian teacher and a Japanese teacher. The following is stated in an interview with an Australian math teacher:

Ms M: Well I have to make sure that everyone's … involved and everyone's … participating in the lesson I guess- that I'm not excluding anyone … that um … that I'm encouraging everyone to have a go …

It seems that she tries to consult with every student individually. On the other hand, the following shows what is typically done by a Japanese teacher during *kikan-jyunshi:*

a. As the teacher looks at the seating plan, he/she identifies which student is doing which type of solution that was anticipated beforehand. (The anticipated solutions including typical incorrect solutions should be created while teachers prepare their lesson plans.)
b. To capture the general tendency of the class.
c. To scaffold and guide "slow learners" if necessary and to provide "fast learners" an extension such as to come up with another way of solving and so on.
d. To plan what he/she should do during the comparison and discussion period.

One challenge for teachers is making anticipated solutions beforehand— this is particularly difficult for novice teachers. Perhaps a Japanese instructor's teaching ability improves as more and more experience in preparing all possible solutions is accumulated. Generally speaking, planning lesson activities is a critical factor in terms of QME. Lesson planning enriches not only a teacher's knowledge, but allows the development of the students' cognitive nature.

During *kikan-jyunshi*, should solutions be reached beyond expectation, the student can be asked to record in detail how the solution was reached. This is also another way of enriching the teacher's knowledge for teaching mathematics. Besides considering individual solutions, it is also important for the teacher to look into the general tendency of the class. In addition, during *kikan-jyunshi*, which student will present first, or where the climax of the lesson should fall all must be taken into consideration. In short, *kikan-jyunshi* provides the teacher with information and time to make scenarios for the discussion-comparison activity. Usually, student teachers or novice teachers find *kikan-jyunshi* extremely challenging to implement in lessons.

Comparing and discussing (Neriage)

During the comparison-discussion period, solutions by the students are presented usually from "basic" to more advanced, or "sophisticated" on the blackboard. Therefore, the teacher needs to be cognizant of constantly selecting only certain students in the beginning of the class versus at the end of class. Some students may wonder why they are always the first person to present.

In Japan, in order to compare and contrast students' solutions, the teacher may carefully plan how the problems will be presented on the blackboard. Japanese teachers use the blackboard to organize both the students' thought processes and their results. Therefore, many teachers in Japan are reluctant to use an overhead projector because it is not suitable for showing the accumulated process of thought.

The comparison-discussion period is also difficult for novice teachers because this activity is closely connected to *kikan-jyunshi*. The novice teachers may give positive comments to each student but struggle to elicit or clarify something the students cannot see. Sometimes, the students cannot distinguish between two similar solutions and the teacher must guide the students to see that these two solutions are significantly different from each other.

The comparison-discussion period is also a place to foster a mathematical way of thinking. This way of thinking cannot be demonstrated simply by using definitions, but rather through problem-solving activities. It is also a place to realize solutions mathematically and gradually allow students to become more intellectually independent. This in turn is the final goal for a Level 3 lesson: human beings may grow together mathematically. Therefore, it is important to recognize the high potential found in each student rather than underestimate each of them.

Sometimes the comparison-discussion activity may appear to be implemented well but in reality that is not the case. There might be a lively discussion between the teacher and the students. Takahashi (2008) described this phenomenon as "show-and-tell." The heart of the comparison-discussion activity is reflective discourse (Doig, Groves & Fujii, 2011). Although rather a classic citation, the comparison-discussion activity is a place to realize Vygotsky's zone of proximal development (Vygotsky, 1978).

Summing up (Matome)
Fujii et al. (1998) argues that most teachers in the U.S. do not incorporate a summing up period, or *matome,* into the end of a lesson. Ending a lesson without summing up leaves the students feeling unsatisfied with what was presented to them. More importantly, the end of a lesson should facilitate a sense of mathematical satisfaction. Summing up a lesson allows the teacher to foster both student satisfaction and mathematical satisfaction. Although it is only done for a short period of time, *matome* is critically important.

Students may listen carefully to what the teacher says as final words. If the teacher concludes the lesson with a procedure-oriented comment, the students may feel that procedural knowledge is the most important aspect of the lesson, or even in mathematics in general. By reminding students how they constructed their ideas by comparing previous knowledge to gained knowledge, the teacher not only applauds the students' thinking, but also reveals the teacher's own value system as well.

Lesson Study for improving QME
Lesson Study Components and its Process
The author identifies five steps in the Lesson Study process (Figure 1). (Fujii, 2014, p.113)

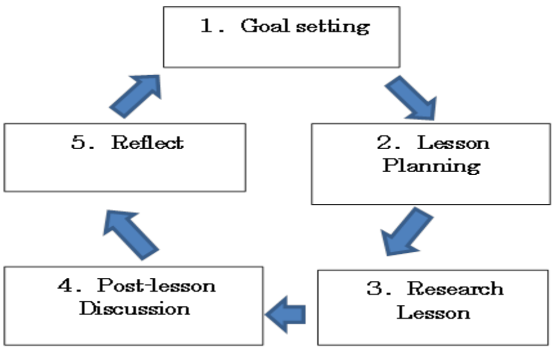

Fig. 1 Lesson Study Cycle (Fujii, 2014, p.3)

Improving QME through Lesson Study
In both pre-service and in-service teacher development programs, Lesson Study plays an important role in supporting teacher growth in Japan. Lectures and workshops, also known as phase 1 professional development (Takahashi, 2011), are effective in increasing a teacher's knowledge for teaching mathematics. On the other hand, Lesson Study, known as phase 2 professional development focuses on developing expertise for teaching mathematics. This includes the expertise needed to develop lessons for particular students with educational aims (Step 1 in Fig.1), to use various questioning techniques, to design and implement formative assessments, to anticipate student responses to questions and tasks (Step 2), and to make purposeful observations during class (Step 3). For teachers to develop such expertise, opportunities to plan lessons carefully are necessary (Step 2), to teach the lesson based on the plan (Step 3), and to reflect upon the teaching and learning based on careful observation (Step 4 and Step 5). Japanese teachers and educators obtain these experiences through Lesson Study.

Unifying mathematics Lesson Study with *the structured-problem solving lesson*

Conducting *the structured problem-solving lesson* in Step 3 of the Lesson Study cycle, the aim of the lesson is to foster the students' mathematical way of thinking. However, thinking itself and the thinking process as a whole are *invisible*. Therefore, it is impossible to teach students how to think directly. Therefore, it is extremely difficult for any teacher to teach students how to think. A teacher can anticipate students' responses beforehand during Step 2, or Lesson Planning. During Step 3: The Research Lesson, the teacher needs to grab *a tail of thought* when it becomes *visible*, and then use this tail as evidence of learning or as a juncture to guide students to further develop mathematical thinking in general. The Research Lesson is an open situation implemented to utilize multiple perspectives with many observers' eyes. During this time, colleague teachers observe what the demo teacher is doing in terms of dealing with students' invisible thinking processes. Following this, Step 4, or The Post-Lesson Discussion, the observers are offered a venue to discuss or evaluate the teacher's behaviors and if they were appropriate or not during the lesson. Finally, the teachers' reflections are written and published as an annual report from the school. Each step in the Lesson Study cycle is closely related. Lesson Study is a system for teachers to learn—how to teach students to become independent thinkers.

Conclusion

Lesson Study can be described as a form of teacher professional development. However Liptak (2002) distinguishes between traditional professional development and Lesson Study in that Lesson Study begins with a question, not an answer prepared by someone else. This difference is the most critical part of Lesson Study. Considering Lesson Study begins with posing a question, it can be said that the first step of Lesson Study is to construct a research theme. When teachers decide on the research theme, they consider the students and their immediate educational goals as well as long-term aims of learning and development. This process needs to be a necessary step in improving QME around the world.

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