Examining the coherence of mathematics lessons in terms of the genesis and development of students' learning goals

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Introduction

This paper aims to clarify the coherent qualities of mathematics lessons referred to as "structured problem solving" (Stigler and Hiebert, 1999). We acknowledge that there is a pattern, or script of mathematics lessons in Japan that Stigler et al. (1999) identified, as we see below. However, we should not directly equate the above teaching pattern with an effective mathematics lesson, because there is a range of teacher efficacy from effective to ineffective, even if the pattern is indeed adopted by most of the primary school teachers in Japan. That is to say, it is not effective to simply use this pattern for teacher development. Rather, it is important to know how lesson coherence can be generated. We believe that there is a substantial difference in lesson quality depending on whether a lesson is developed like a narrative or in isolated steps. From this perspective, we identified a narrative plot as coherent accounts of a sequence of events and activities in the lesson produced by an excellent teacher (Okazaki, Kimura, and Watanabe, 2014).

In this paper, we analyze lessons given by two experienced teachers to further clarify the coherent quality of a lesson by focusing on the genesis and development of students' learning goals.

Theoretical background

We hypothesize that children's learning is narrative in nature, and that a quality lesson is developed in a narrative form. For instance, Dewey (1915, p.141) stated , "Its intellectual counterpart is the story-form…(Children's) minds seek wholes, varied through episode, enlivened with action and defined in salient features…Analysis of isolated detail of form and structure neither appeals nor satisfies." This suggests that even if we collect all of the parts that constitute a lesson structure, the lesson will not attract the attention of children unless it is in story form. We consider the concept of plot as being crucial to a quality lesson. Krummheuer (2000) stated that "a plot characterizes the sequence of action in its totality: it describes something that is already fixed… But an unfolding plot connotes something fragile, not yet entirely executed, still changeable." (p. 25).

We consider the following script (J script), identified as a Japanese lesson structure (Stigler and Hiebert, 1999), as a way of adding the role of the plot to a narrative structure.

•Reviewing the previous lesson;
•presenting the problem for the day;
•students working individually or in groups;

•discussing solution methods; and
•highlighting and summarizing the main point.

However, even if teachers use this pattern in their teaching, there is a range of possible lesson evaluations from very good to very poor. Thus, it is important to examine how a coherent plot in teaching can be produced during a lesson. In particular, we should note that the protagonists are the students, and therefore, their awareness of problematic situations is a central component of the story.
What children explore in a mathematics lesson should be distinguished between a problem and a task. In Japanese lesson, while a problem is presented by a teacher, a task is collaboratively set by a teacher and children according to what they recognized as problematic in trying to solve the problem. We believe that a task needs to be the children's own learning goals when the lesson is narratively coherent.
Moreover, we assume that learning goals develop during a lesson. Koto and Ikeno (2010, p.16) distinguished four levels of learning goals by focusing on children's problem solving: Idea, Process, Relation, and Effectiveness. Our focus, however, is not solely on children's problem solving, but on what process children can use to construct mathematical concepts by reflecting on their problem solving. We thus hypothesize the following levels of the learning goals that permit us to explore the coherent quality of a mathematics lesson starting from students' problem solving in order to construct mathematical concepts.

Level 1 (L1) (Idea level): Children aim to develop their own idea for attaining an answer.
Level 2 (L2) (Process level): Children aim to examine the adequacy of several methods for solving the problem.
Level 3 (L3) (Argumentation and effectiveness level): Children aim to explain why the methods are valid or to provide a more effective explanation based on the known solutions.
Level 4 (L4) (Concept level): Children aim to clarify a new mathematical concept by reflecting on their previous inquiry processes.

Below, we will use the labels (Ln) to characterize the levels of the students' learning goals.

Methodology

We asked six teachers to conduct lessons: A) two experienced teachers who specialize in teaching mathematics; B) two experienced teachers who do not specialize in teaching mathematics; and C) two teachers who have a few years' experience teaching. We selected the content 'area of a parallelogram for which the height cannot be known from a straight line on its inside' from a fifth grade mathematics textbook (Fig.1, right).

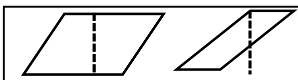


Figure 1.

During a preliminary meeting with each teacher, we asked the teacher to conduct their lesson to help students find multiple solutions and to understand the concept of area beyond simply teaching how to solve the problem.
The lessons were recorded by video cameras and field notes. We made transcripts of the video data. In our data analysis, we first extracted all meaningful episodes to examine how a teacher interacts with students. Next, we conceptualized each episode unit in

terms of the interaction's practical effects, before reconstructing the entire picture of the lesson structure, i.e., the 'plot'. Finally, we compared the reconstructed lesson structures and attempted to clarify the characteristics of coherent plots. Below, we present our analysis of the lessons conducted by two experienced teachers: Mr. F (the above type A) and Ms. Y (type B).

Results

The case of Mr. F's lesson

First scene: Reviewing the formula for the area of a parallelogram

Mr. F began by reviewing the formula for finding the area of two parallelograms (base 6, height 4; base 3, height 1) (Fig. 2). Here, in the order given, he interacted with the students: 1) asking the students for the value of the area by counting the unit squares or by using the area formula; 2) asking what formula they used; 3) asking what 6 and 4 in the formula referred to in the figure; and 4) checking the vertical relationship between the base and height and the arbitrary placement of the height of the shape. This pattern of interaction was also used in the case of the 3×1 formula.

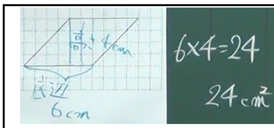


Figure 2

Second scene: Setting a problem through an experience of conflict

Mr. F presented a problem as follows.

Mr. F: I have one issue with this. I am bothered by this parallelogram. Do you understand my concern?

Student 1: The previous parallelograms had this line. This time, we can't draw this (line) (Fig. 3).

Mr. F: I tried to find the height, but there's nothing there! Oh, there's no height!

Students: But… (Several students raised their hands to respond.)

Mr. F: But, does the parallelogram have an area?

Students: Yes, it has an area.

Mr. F: Yes, it does. This is a parallelogram. But we can't use the area formula because we don't know the height. Don't you feel like crying?

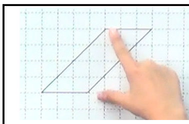


Figure 3

The problem setting was like the beginning of a narrative in which the students were involved in an issue troubling Mr. F, where the two circumstances ("there is no height" and "the area formula can't be used") were given as the problematic aspects of the issue.

Third scene: Setting a goal by comparing the known and unknown

Mr. F proposed setting a learning goal. He confirmed that the height is problematic and tried to direct the students' interest to figuring out a formula of a parallelogram of unknown height. Moreover, he clarified the task by aligning three parallelograms and confirming that the formula could now be used only for 6×4 and 3×1 parallelograms (Fig. 4). Students could then set a goal: to find the area of a parallelogram of unknown height using a formula. We note that this "aligning" implicitly prepared the students with insight for seeing the parallelogram as half of a 6×4 parallelogram and as four 3×1 parallelograms.

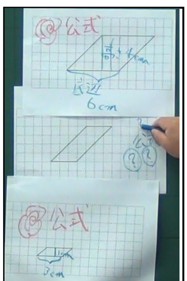


Figure 4

Fourth scene: Individual activities and redefining the goal

The students individually tried to solve the problem. However, Mr. F found that some students just wrote the formula 3×4=12 procedurally (L1), which was different from the set goal. Mr. F then stopped them and restated the task to all the students.

> Mr. F: … As it is now, we don't know whether this is the height or not, because it doesn't meet the base. So, you can't set this as the height (Fig. 5). Consider using the formulas you already know.

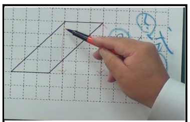


Figure 5

Mr. F's redefining of the goal in this way seemed to work successfully because all the students then began considering the problem using the known parallelograms.

Fifth scene: Class discussion (1) — Sharing the fundamental idea

We found that Mr. F employed one particular type of interaction in which he attempted to deepen one basic idea by using plural voices (Fig. 6). First, Mr. F invited the students who had come up with the idea of using four 3×1 parallelograms to present their idea to the class (L3). Mr. F's writing on the blackboard gradually became more detailed as he interacted with the different students. We characterize this series of interactions as multi-layered.

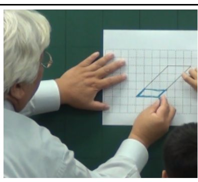
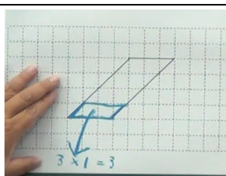
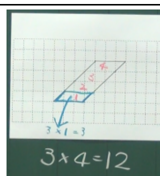
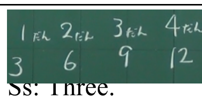
	 llelo- there er is else explain in the same way? (He wrote the formula.)	 m ere ur , 2. (He wrote the numbers.)	 Ss: Three. Mr. F: What about for two steps? Ss: Six. …

Figure 6

Sixth scene: Class discussion (2) — Sharing various ideas

Mr. F invited the students to share their other ideas, and the four ideas were presented (Fig. 7). Here, how to transform the parallelogram into known figures and the relevant formulas were confirmed (L2).

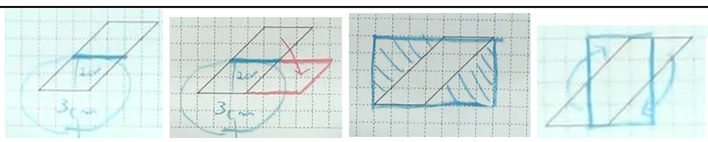


Figure 7.

Seventh scene: Class discussion (3) — Rethinking the goal

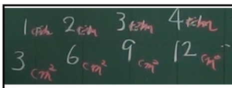


Figure 8

Mr. F proposed a rethinking of the main goal, and asked the students again what the height of the shape was. The students were not confident in their answer. Here, Mr. F told them to reflect on the idea shown in Figure 6, saying together with the students, "The height of the smallest one is 1 cm, the height of the parallelogram one step higher is 2 cm…" (L3). Moreover, he modified the table by changing the word "step" to "cm" and newly adding cm^2, indicating the area of each smaller shape (Fig. 8). As a consequence, the students could reinterpret one "step" as 1 cm of height and then understand that the area formula that they already knew was actually applicable to all parallelograms (L4).
The lesson ended by applying the formula to other figures and summarizing the main points of the lesson (The final, eighth scene).

The case of Ms. Y's lesson

First scene: Presenting the problem and setting the learning goal through discussion and a student's question

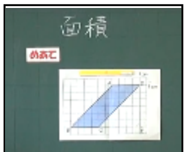


Figure 9

Ms. Y set the problem of "finding the area of parallelogram with the base BC" (Fig. 9). Several students voiced their question "Does BC need to be the base?" Ms. Y let them discuss the question in small groups. One student M asked "Can we count the number of unit squares?" Ms. Y halted the discussion and attempted to set the learning goal using M's question.

> Ms. Y: M asked if we can count the unit squares or not. What did you learn yesterday?
> Students: Base times height. It's the formula for a parallelogram.
> Ms. Y: Use this, because you learned it.

Moreover, she added a task of thinking about how to explain their ideas.
In the second scene (individual activities), the students attempted to find their own solutions.

Third scene: Presenting and sharing ideas and enhancing the task consciousness


Figure 10

Ms. Y asked student M to present his idea of dividing the parallelogram into top and bottom parts and arranging them alongside. Here, we could observe a pattern of interaction. First, student M stated the formula and the answer. Next, Ms. Y demonstrated his idea by using paper that she had already prepared (Fig. 10). She then asked the other students to explain M's idea empathically.

> A student: I think that he considered that the figure can't become a rectangle even if cutting it vertically. He changed it into a parallelogram with a shape like rectangle.

Ms. Y again confirmed the formula 8×3=24, i.e., she encouraged the students to share the idea by reproducing the idea and involving other students (L2). We could observe this interaction pattern repeatedly. Here, student K questioned M's idea, because it violated the condition that BC was the base (L3). The pros and cons of opinions were stated. Ms. Y then clarified the meaning of the two opinions, and left the situation unsolved.

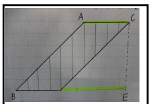


Figure 11

Next, student S stated his procedure of multiplying the base by the height (Fig. 11), and emphasized that the height was the distance between parallel lines (L1). Ms. Y asked all students, "Why can we consider the height of the parallelogram to be a line from a vertex to a line that is the extension of the base?" Here, Ms. Y did not authorize his procedure. We believe that she envisaged that the students may have further learning goals.

<u>Fourth scene: Exploring new ways to set the height inside the figure</u>

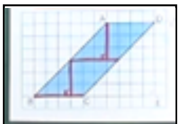


Figure 12

Ms. Y asked another student R to present his idea of dividing the parallelogram into half but without moving the figure (Fig. 12), and let the students consider the difference between the ideas of M and R, by which she envisaged that the students would notice the height was the sum of the heights of two parallelograms. Here, Ms. Y wrote the formula 4×3+4×3, transformed it into 4×(3+3), and showed that the height was 6 (L3).

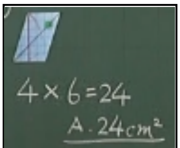


Figure 13

Finally, Ms. Y asked the student KN to present his idea of isometric change by cutting the diagonal of the parallelogram (Fig. 13). The students accepted the idea since the height appeared inside the figure, and the formula became 4×6, which was same as the above formula (L3). Moreover, Ms. Y attempted to integrate the previous ideas, saying that KN's idea was the same as M's idea because the height appeared inside the figure and that it was also the same as S's idea because the height was the distance between the upper and lower bases.

The <u>fifth scene</u> was a summary of the main points, which was followed by solving the application problems in the (final) sixth scene.

Discussion

We regard that both Mr. F and Ms. Y's lessons provided coherent practices because the students in each class developed their learning goals through each lesson. We first compare the two lesson scenes with the J script. We find there are the scenes (<u>underlined in Table 1</u>) specific to Mr. F and Ms. Y, which do not clearly correspond to the J script. We consider these scenes and activities crucial in making a lesson coherent. The first characteristic is that both teachers set the initial learning goal after evoking students' cognitive conflicts or their own questions. The second characteristic is that both teachers enhanced the learning goal in the middle scenes of the lesson, as Mr. F asked the students to rethink the meaning of height and Ms. Y set a new goal of creating an idea that connected the underlying solution and the area formula.

Let us conceive the lesson coherence in terms of the levels of learning goals. It is then found that students in both classes at least reached L3, since they examined why the height was the distance between the upper and lower bases beyond simply finding solutions to the problem. The difference between lessons was how the students understood the height. The students in Mr. F's class understood the height to be the number of parallelograms of 1 cm in height (L4) while the students in Ms. Y's class understood the height to be simply the length between parallel lines. We believe that Mr. F's mathematical knowledge raised the learning goal to L4.

Table 1. Comparison of lesson structures among Stigler et al., Mr. F, and Ms. Y.

Mr. F	Stigler et al.	Ms. Y
1. Reviewing the formula for the area of a parallelogram 2. Setting a problem through an experience of conflict 3. Setting a goal by comparing the known and unknown 4. Individual activities and redefining the goal 5. Sharing the fundamental idea 6. Sharing various ideas 7. Rethinking the goal 8. Applying the formula to other figures and summarizing the main learning points of the day	1. Reviewing the previous lesson 2. Presenting the problems for the day 3. Students working individually or in groups 4. Discussing solution methods 5. Highlighting and summarizing the main point	1. Presenting the problem and setting the learning goal through discussion and a student' question 2. Individual activities 3. Presenting and sharing the ideas and enhancing the task consciousness 4. Exploring new ways to set the height inside the figure 5. Summarizing the main learning points 6. Solving the application problems individually or in small groups

We note that Mr. F and Ms. Y commonly used a pattern of interaction of clarifying the meaning of an idea by connecting students' plural voices.

In summary, we conclude that a lesson can become coherent when the students' learning goal gradually develops in terms of its level and when there are fruitful interactions for connecting and reflecting on ideas, beyond simply following the J script.

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