Conceptualizing quality mathematics education

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There is no doubt that education is aimed to convey the culture, to facilitate the societal development and to improve the quality of citizens' life. However, the quality of education might differ between cultures and societies, and hence, the development of a society and the citizens' vision of life will be different. The same exposition works in the topic of quality mathematics education (QME). In the following, we elaborate on the societal differences between the Western and the Eastern cultures and how their directions for improving the quality of mathematics education are supposed to be different. We start discussing the Western situations first and then the Eastern situations as introduction to the discussion on the Taiwanese case as an example to particularize its reform of mathematics education at present.

The improvement of mathematics education in the Western countries

In this section, we describe the societal foundation in most Western countries, and then focus on their latest efforts in advancing mathematics education.

The foundation of education—Equity

In the Western societies, especially the Europe, *equity* is the most natural issue because of their well-developed welfare system and emphasis on human rights. As a result, the unjust concern is relatively rare when compared to their counterparts.

The regional collaborative project—Inquiry-based learning

Regarding the research issues popular across countries in the recent decades—the inquiry-based learning and teaching dominates. There are many research projects emerging from the problems of mathematics teaching and learning. For example, one commonly discussed research project in the Europe is the application of inquiry-based learning (IBL) in classroom practice. The purpose of IBL could provide opportunities for students to practice exploring, conjecturing, reasoning logically and evaluating in various situations in mathematics learning (NCTM, 1991), and this intention of IBL is very close to the idea of improving students' mathematical competencies in the Programme for International Student Assessment (PISA). In the special issue of the ZDM in 2013, the advantages of IBL in mathematics teaching and learning and their importance in present education are discussed in a wide range of research projects across the European countries.

However, the implementation of IBL challenges many countries. For example, in the USA, various educational systems and the high-stakes tests make IBL difficult to work. After the establishment of the new curriculum CCSS-M, the USA tries again to clarify the societal needs, schooling traditions, the new curriculum goals, and the readiness of teaching (Schoenfeld & Kilpatrick, 2013), to ensure the possibility of IBL working in the classes. In Europe, many countries face the main problem of practicing teachers' beliefs and practices and how to evaluate the intervention becomes a difficult task (Engeln, Euler, & Maass, 2013).

The goal of QME—Mathematical literacy
For most Western countries, mathematical literacy is a significant goal of education for students because it recognizes the increasing importance of mathematical proficiency in the modern world. Hence the large-scale international assessment PISA affiliated with OECD focuses on mathematical literacy (see Stacey & Turner, 2015). Moreover, many countries develop their national curriculum under this concept. For instance, in *Germany*, mathematical literacy is emphasized in the national curriculum. The national standards of mathematics is therefore designed based on the different strands of *competence*, e.g. problem solving, argumentation, communication, modeling, etc., different levels of *requirements* to achieve those competencies in different *content areas* as the structure of mathematics curriculum (KMK, 2003). The educational policy following this orientation of curriculum can therefore provide a clear structure for teachers to teach and evaluate students' performances with clear rubrics. In *England*, the curriculum goals are presented as dealing with three essential issues, the fundamental knowledge for students to learn hierarchically, the thinking and reasoning processes to experience from the beginning exploratory activities to advanced mathematical proof, and the application of what students' learned in dealing with various problems (Department of Education, UK, 2013).

Quality mathematics education in the Eastern countries—The case of Taiwan
There is one significant phenomenon of students' performances in the international large-scale mathematics assessments (e.g. TIMSS and PISA) in the East Asia (Lin, 2007). The Asian students show a regular combination regarding their mathematics achievement and learning attitude, which could be represented in two groups. One is the group whose students are with high achievement but low self-confidence, such as Hong Kong, Japan, Korea, and Taiwan. The other one is the group whose students with low achievement but high self-confidence, such as Indonesia, Malaysia, and the Philippines. However, the factors in causing such phenomenon have not yet been established. Nevertheless, the ecological distribution of those countries shows two aggregations. Hence, the effort to improve QME might also be different from the Southeast Asia and the Northeast Asia.

Take Taiwan as an example in improving the quality of mathematics education. There are three emergent demands need to be answered, and therefore the policy of reform should correspond to those demands, which are discussed as follows.

Three demands for reform
<u>Developing fair learning opportunities for students</u>
Though the Taiwanese students outperform their counterparts in international studies on mathematics, the high achieving groups and the low achieving groups present a severe polarization that reveals a serious mathematics educational problem in Taiwan. The below-average students lack meaningful opportunities to learn mathematics as compared to the above-average students.

<u>Motivating students to think and engage actively</u>
There is also one learning problem in Taiwanese education. No matter how varied the achievements of students are, they are learning passively. Such passive attitudes might be due to the examination culture dominating students' lives in Taiwan (Lin & Tsao, 1999).

Cultivating students' learning power

Lin (2012) proclaims that enhancing the *learning power* (cf. Crick, Broadfoot, & Claxton, 2004) should be a major goal in the latest educational reform in Taiwan. He defined that the learning power is supported by three fundamental component sets of learners' *tools* (including language and thinking), *learning methods* (including reading and inquiry), and *dispositions* (including learners' emotions, attitudes, and beliefs).

Concerning these demands in learning school mathematics, the Ministry of Education (MOE) has started to initiate a two-stage reform to overcome this challenge for the improvement of the quality of mathematics education. The first stage provides an opportunity for mathematics teachers to develop their professions with professionals, e.g. their peers, colleagues and mathematics educators. The second stage emphasizes the design and implementation of foundation construction activities for students. The actions regarding these two stages are presented.

Actions on improving QME

The stage of the creation of the wow factor for mathematics teachers

In order to create the wow factor for schools, the reform in the first stage steers the teachers' professional development. The National Team for Educating In-service Teachers has started the Lighten-Up School-Based Program (LUSBP), a design-based research project with the rationale of providing student-centered instruction since 2012 (Lin, Hsu, & Chen, under review). Seven innovative teaching themes in mathematics are involved. They are mathematical conjecturing and argumentation, diagnostic teaching, mathematical literacy and assessment, mathematical modeling, ICT, inquiry-based teaching, and reading comprehension. The LUSBP matches university (teacher) educators with mathematics education research background, who then visit elementary and junior high schools to provide professional development workshops six times a semester for in-service mathematics teachers on a specific theme. At the same time, these teacher educators also deal with their professional learning and development through the workshops. The specific feature of this program is that students, teachers, and educators all act as learners. So far, there are participants of elementary schools, junior high schools, and educators in numbers of 30, 22, 47; 29, 34, 51; 24, 35, 40, and; 41, 27, 52 in semesters of school year 2012 (1&2), 2013 (1), 2013 (2), and 2014 (1). More than 90 percent of mathematics educators in Taiwan participated.

The stage of the design and implementation of foundation construction activities for students

For students' passive learning attitudes and the deficient learning activities in school mathematics in Taiwan, the reform in the second stage concentrates on providing a variety of learning activities for students to learn mathematics, called *foundation construction* project, starting in 2014. Four different actions are planned for this project: (1) designing activities deeply; (2) implementing activities nationally; (3) providing mathematics activity camps for students, at least half of whom are low achievers, and; (4) building mathematics diagnostic systems with enrolled volunteers.

The activity design of this stage underlines a *structuralist approach* to designing embodied tasks for students to engage. Moreover, the activity design also applies a *neo-diagnostic conjecturing* approach, which means students' learning difficulties can be diagnosed during their engagement in the activity before the regular lessons, rather than receiving additional remedy in teaching for their deficient learning. Therefore, the *design* of foundation construction activities becomes relatively important before its implementation. The characteristics of designing the foundation construction activities can be condensed in two. One is that the activity should be designed as a *spiral* diagnostic activity for different age groups. The other one is that students' *thinking pattern* is the pivotal line for developing the activities. This means the activities should be designed using students' misconceptions as the starting point (Piaget, Inhelder, & Szeminska, 1960).

The activity *designers* should design activities toward the abovementioned two specific characteristics. These designers should then deliver the rationale and instruction of their designed activities to the activity *spreaders* who are from different schools in order to implement the activities in workshops. These spreaders will generate a new set of spreaders when they back to their schools. Working in several cycles, those activities can be spread out and implemented over schools in Taiwan. Each activity is expected to train at least 2 spreader teachers in each school. There are around 3,500 elementary and junior high schools in Taiwan. Therefore, around 7,000 spreader teachers will be generated in delivering each activity in schools. With the expectation that these teachers can train a new set of spreaders in schools in two years, the number of spreader teachers might ultimately reach the 20,000 to 30,000 for each activity in Taiwan.

In addition to those training for teachers, the plan is to also provide weekend, winter and/or summer *mathematics activity camps* run by activity spreader teachers. The attendance of those students should cover at least half of the low achievers in each school. Last but not least, the center of mathematics education at the National Taiwan Normal University also plans to build a mathematics *diagnostic system* run by enrolled volunteers who are retired or experienced teachers and scholars with strong mathematics or mathematics education background. The teaching contents of this system focus on *axial* mathematics topics instead of fragmentary contents.

Conclusion

In summary, the state of QME in Taiwan can be viewed from three phases: the educational rationale, the learning & teaching approach, and the goal of ME. The considerations from other East Asian countries will be extended in the panel discussion.

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