Assessment of secondary students' mathematical competencies

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Introduction

There is an urgent need to equip young learners with the necessary skills and knowledge in mathematics for the future. In mathematics education, mathematical modeling has increasingly becoming more relevant in developing students' functional competencies required in the 21st century. Students not only need to acquire functional skills such as problem solving and reasoning but the more important question is how to use them in life (OECD, 2009). Mathematical modeling provides a platform where this knowledge can be utilised in building a model to solve real-world problems. This approach is very valuable in the teaching and learning of mathematics (Blum, 2011).

This paper describes the assessment on the modeling competencies of two groups of Form 4 (16 year old) students in Malaysia. The analysis covered the mathematical competencies of identifying variables, making assumptions, mathematics reasoning and interpreting solutions. In addition, this study also reported the challenges and the difficulties students faced when solving the modeling task.

Literature review

Mathematical modeling is the process of translating a real-life problem into a mathematical problem (Ang, 2001). Solving the modeling problem might not be simple as it usually involves integration of a few mathematical concepts (Ang, 2009). According to Blum (2011), mathematical modeling consists of tasks that require the translation between reality and mathematics. Mathematical modeling allows students to experience mathematical situations in real life (Pollak, 1979). A real modeling task would change a person's view on mathematics as a precise and accurate field to understanding it as having imprecise estimations in reality.

A simple mathematical modeling process consists of four modeling stages, namely, *Observation, Analysis, Interpretation and Application* (Ang, 2001; Swetz & Hartzler, 1991) although the terms used may differ according to researchers. Any modeling process begins with the real world problem that can be formulated into a mathematical problem. The mathematical solutions obtained are usually interpreted in the real-world context before they can be accepted.

One of the important goals of mathematical education is the development of students' mathematical modeling competencies (Chan, Ng, Widjaja & Cynthia 2012). Such development depends on the modeling perspective and the goals intended to be achieved. Most definitions of mathematical competencies involve mathematising the problem and formulating models during the modeling process. The Program for International Students Assessment (PISA) (OECD, 2010) regards modeling competency as a part of mathematical literacy. For a student to be proficient in mathematics, it is not necessary for students to go through every stage of the modeling process.

According to Niss, Blum, and Galbraith (2007), mathematical modeling competency requires the ability to identify the variables, make suitable assumptions, mathematising the real-world problem and interpreting and validating the solution. However, Maass (2006) defined modeling competencies as "skills and abilities to perform modeling process appropriately and are goal oriented as well as willingness to put these into action" (p.117). Jansen (2006) described it as a person's readiness to do something in response when given a mathematical challenge in a situation. Assessment of mathematical modeling competency can be done using the multidimensional approach (Jensen, 2006). This multidimensional paradigm consists of degree of coverage, radius of action and technical level.

Methodology

Sample and location

This study was carried out in a private secondary school in Malaysia. Fifteen students from a mixed-ability class in Form Four 4 (Grade 10) were involved in the research. They worked in groups of three or four. The students had some experience with modeling tasks as the modeling lesson was taught by their mathematics teacher.

Modeling Task

The modeling task was designed by the authors based on the 7-step modeling process by Galbraith (1989, 1995) Figure 1). These steps are an elaboration of the simple mathematical modeling process. In this modeling task, students were required to estimate the maximum height of a building that can be reached by the fire engine ladder. This task was also piloted with a few students to reveal their understanding of the questions in the modeling task. The teacher also went through the whole modeling task with several colleagues to gather feedback that can be used to refine the task.

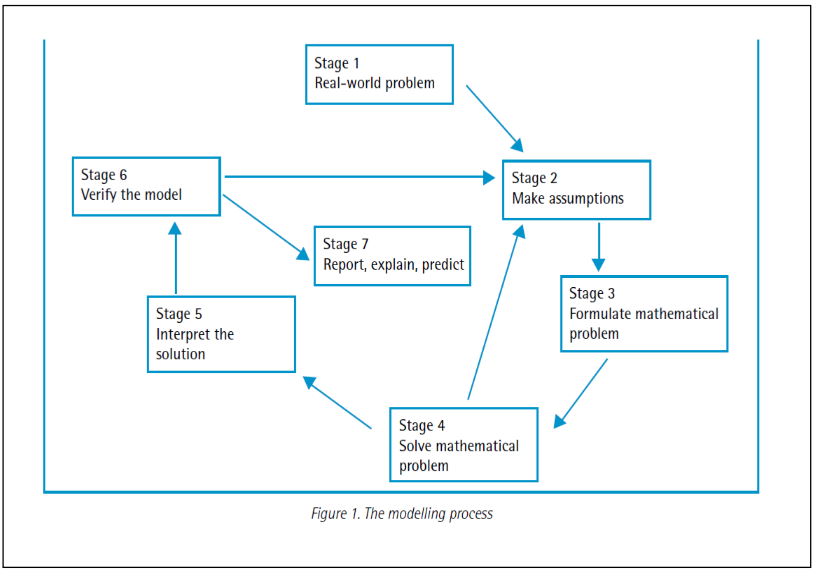

Figure 1. Mathematical modeling cycle process by Galbraith (1989, 1995)

Data collection and analysis
This is an exploratory qualitative analysis of students' responses based on the modeling task developed by the researchers. The students worked in groups to solve the task. In addition, students' responses were also analysed to assess their modeling competencies.

Assessment of students' mathematical competencies
In developing the competencies criteria, several dimensions of modeling competencies mainly from the modeling process by Galbraith (1995) were taken into consideration. In addition, the exposure of these modeling tasks to students for only a short period and students working in groups were also considered. This study focused on the elements of mathematical competencies such as understanding, simplifying, formulating, solving and verifying. The three modeling competencies of making assumptions, computing and interpreting solution and mathematical reasoning are assessed using the rubric developed. This rubric rates the students using the 4-point scale from unsatisfactory to distinguished as shown in Table 1 (next page). This section discusses the assessment of two groups of students concerning their mathematical modeling competencies using the assessment rubrics in Table 1. The assessment was carried out by investigating students' written works and obtaining their responses through interviews.

Exemplification of Band 2 mathematical modeling competencies (Group A)
Group A students were assessed to be in band 2 as they used the stability of a ladder to find the maximum height of a building that can be reached with the fire engine ladder. There are several aspects that show this group of students had difficulties in managing real-world problems from the data collected.

<u>Competence in making assumptions</u>
These students made the assumptions that the farther the distance between the fire engine's ladder and the building the more stable the ladder (see Figure 4). This shows that the students took the safety of the firemen into consideration when finding the maximum height of the building. Although they considered the stability of the ladder, they did not consider its length and the limited area for a fire fighting car to park. The second assumption was the farther the distance between the fire engine's ladder and the building the higher we get to the building. This would place the ladder nearer to the highest point of the building. This assumption is wrong.

<u>Competence in computing and interpreting of solution</u>
Generally, the students had difficulties in understanding the problem statement given; they could not list out the important keywords from the statement or even restate the problem. These difficulties led to their misinterpretation of terms and inability to solve the problem. Based on the responses, the students believed the problem was to estimate the height of the building for the fire engine's ladder to reach it but they overlooked the point about the length of the ladder. The students did not elaborate on the stability and material of the ladder. When the group was discussing, no one brought up the issue of what they must look for to maintain the stability of the ladder. For instance, the angle of elevation of the ladder could be the factor. In this case the students were unable to make an assumption based on the problem situation involving the length of the ladder and the height of the building.