Student perspectives on the factors of being a great mathematics teacher regarding teaching methods

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Introduction and significance

Chinese teaching has been labeled as teacher-led, and the students are considered to learn mainly by listening to teachers (Wong, 2004). Based on this perspective the ability to create a teacher-led learning environment becomes a criterion for a "good" teacher in Chinese society. A crucial factor for creating a teacher-led environment is the ability to explain concepts clearly (Leinhardt, 2004). The findings of Kaur's studies (2008, 2009) revealed that Singaporean students considered mathematics teachers as satisfactory if they could explain difficult content clearly and render it easily understandable. Kaur's findings matched the results of studies in Western societies, such as Murray's study (2011) in Australia, and Lavy and Shriki's study (2008) in Israel.

The characteristics of great mathematics teachers have generated substantial attention from researchers (Cai, Perry, Wong, & Wang, 2009; Hsieh, 2012). Researchers attempting to conceptualize great mathematics teachers have endeavored to investigate the characteristics of great mathematics instruction (Kaur, 2009); specifically, some have focused on the teaching methods (e.g., techniques, activities, devices, and schemes) that a great mathematics teacher should implement (e.g., Anthony & Walshaw, 2009; Lin & Li, 2009).

Curriculum reforms in Chinese societies such as China and Taiwan have adopted Western student-centered views (Ding & Wong, 2012, Hsieh, 1997). Educators and teachers were encouraged to use innovative teaching methods such as hands-on investigation to achieve the reform goals. However, to implement the student-centered approach, students' perspectives of effective teaching methods should be seriously considered. This study is the first to collect Taiwanese students' perspectives on this topic.

Research purpose

The research questions of this study were as follows:

1) What are the teaching methods that a great mathematics teacher would use in the classroom in Taiwan? How crucial are they?
2) What factors contribute to being a great mathematics teacher regarding teaching methods from students' perspectives?
3) Do middle school and primary school students have different perspectives on factors contributing to being a great mathematics teacher? What are the differences?

Research design

This study was conducted in two stages. In the first stage, a qualitative pilot study employing open-ended questions was conducted. A total of 238 high school students participated in the first stage. The students were asked to provide their opinions regarding what a great mathematics teacher would do when performing a variety of teaching tasks such as introducing new mathematical concepts.

A content analysis of the students' responses was performed to obtain dimensions and items related to mathematical teaching competence. Experts that include university mathematics educators and researchers, school-based supervisors of prospective mathematics teachers, and expert school mathematics conducted the content analysis. A literature review was conducted to obtain additional items (e.g., "present incorrect ideas or solutions for us to figure out") that were not revealed explicitly in the students' data. The dimensions and items obtained in the first stage were used to develop the instruments for the second stage of the study.

In the second stage, two questionnaires with dichotomous items were developed. One questionnaire was for the middle school (and high school) study; the other was for the primary school study. The items in the two questionnaires were identical[1] and obtained from the first stage of the study. The questionnaires were distributed to nationwide random samples of 1,040 primary and 1,039 middle school students. In the questionnaires, students were asked to state whether a great mathematics teacher should focus on the described behaviors in a variety of teaching contexts. This paper discusses the items in the dimension of mathematics teaching methods. This dimension contained 16 items. The stem and list of items and their codes used in this paper are shown in Table 1.

Table 1. Items relating to mathematics teaching methods

When teaching us mathematics, a great mathematics teacher should….

□ Present incorrect ideas or solutions for us to figure out. (TD01)
□ Provide hands-on activities for us to understand mathematics in class. (TD02)
□ Ask us to solve problems on the board to enable us to learn how others solve the same problems. (TD03)
□ Provide opportunities for discussion. (TD04)
□ Use simple and clear words to introduce new ideas. (TD05)
□ Pose questions and ask us to answer them to enable us to learn what the teacher is teaching. (TD06)
□ Guide us in observation and induction to develop our concepts. (TD07)
□ Before explaining to us, ask us to guess possible answers to some problems in class. (TD08)
□ Emphasize critical ideas repeatedly in class. (TD09)
□ Lecture mainly to avoid unnecessary wasting time. (TD10)
□ Explain to clarify our confusion and doubts. (TD11)
□ Ask us to explore new ideas before providing instructions. (TD12)
□ Allow us to learn through games in class. (TD13)
□ Introduce new concepts from easy to difficult levels. (TD14)
□ Employ small group learning when applicable. (TD15)
□ Switch between various teaching methods rather than use only one method. (TD16)

Data analysis

This study performed several exploratory factor analyses (EFA) with oblique rotation on the data from all students to determine the factor structures of the scale of teaching methods. The EFA were conducted using M-plus 6.12 with a robust weighted least squares estimator that is considered robust to non-normality (Flora & Curran, 2004). Several criteria were considered to determine the number of latent factors extracted, including a comparative fit index (CFI) higher than 0.90, a Tucker–Lewis Index (TLI) higher than 0.90, and a root mean square error of approximation (RMSEA) lower than 0.08 (Fan, Thompson, & Wang, 1999; Hu & Bentler, 1999; Kline, 2011). In addition, the number of eigenvalues greater than 1 was examined according to the Kaiser–Guttman rule.

The structures identified by EFA were then tested using a series of confirmatory factor analyses (CFA) with a robust weighted least squares estimator and factor variances fixed at 1 in M-plus 6.12. The structural relationships between the latent constructs with combining or separating data from the two school levels were examined. Model fit statistics including CFI, TLI, and RMSEA were used as mentioned previously.

Descriptive information including the percentage of checks (POCs) for each item and the weighted average POC for each latent factor was computed. To obtain the weighted average POC for each factor, each student's POCs of all indicators were weighted using the factor loadings of the indicators as weights, and the weighted POCs were averaged over all indicators in the same factor. The weighted average POC for a factor was obtained by calculating the average over the weighted average POCs of the relevant student cohorts.

Research results

The mathematics teaching method was determined using three latent factors, each of which comprised five indicators from the EFA results (TD16 was deleted after the EFA). Table 1 shows the indicators of the three factors and their factor loadings obtained from the EFA.

The first factor was named *question-heuristics* and relates to asking students questions, explaining, and requiring students to guess or figure out others' solutions. The second factor was named *long-term task* and relates to arranging investigative tasks that require time to complete, such as games, small group discussion, and hands-on activities. The third factor was named *idea illustration* and relates to teaching from easy to difficult ideas, frequently repeating crucial ideas, and clarifying students' doubts.

The final CFA tested separate models, one for the middle school level and one for the primary school level. The structural relationships among *question-heuristics*, *long-term task*, and *idea illustration* demonstrated good fits to the data (middle school level: CFI = .954; TLI = .944; RMSEA = .038. primary school level: CFI = .960; TLI = .952; RMSEA = .035).

Table 1. Pattern coefficients of the teaching method

Items	Question-heuristics	Long-term task	Idea illustration
Present incorrect ideas or solutions (TD01)	0.779		
Mainly lecture (TD10)	0.488		
Ask students to guess before explaining to them (TD08)	0.440		
Ask students to solve problems on the board (TD03)	0.414		
Pose questions and ask for answers (TD06)	0.389		
Allow students to learn through games (TD13)		0.890	
Employ small group learning (TD15)		0.790	
Provide opportunities for discussion (TD04)		0.668	
Provide hands-on activities (TD02)		0.615	
Ask students to explore new ideas (TD12)		0.344	
Teach from easy to difficult levels (TD14)			0.862
Clarify students' doubts (TD11)			0.754
Guide students in observations and induction (TD07)			0.694
Emphasize crucial ideas repeatedly (TD09)			0.593
Use simple and clear words to introduce ideas (TD05)			0.450
Percentage of variance accounted	41.09	10.58	9.24

Note. CFI = 0.989, TLI = 0.982, RMSEA = 0.022.

Figure 1 shows that the three factors were correlated mutually at each of the school levels. At the primary school level, the correlation between *question-heuristics* and *idea illustration* was high, with a coefficient of 0.90. This may indicate a requirement of a great mathematics teacher rated by students, that when illustrating an idea through observation or clear direction, questioning should be used often.

Although the primary and middle school levels share the same structural models, the relative sizes of the factor loadings for the indicators are not identical. The most distinct size appears in the *question-heuristics* factor, of which indicator TD08 has the highest factor loading at the middle school level but is only at the middle rank of the factor loadings at the primary school level. In the *idea illustration* factor, the ranks of the indicators for both primary and middle school levels are identical.

For each factor, the weighted average POCs were statistically significantly different between school levels (see Table 2, Figure 2). The Cohen's d values between the school levels showed that the school level has a medium to low effect on the *long-term task* and *question-heuristics* factors, and has no notable effect on the *idea illustration* factor.

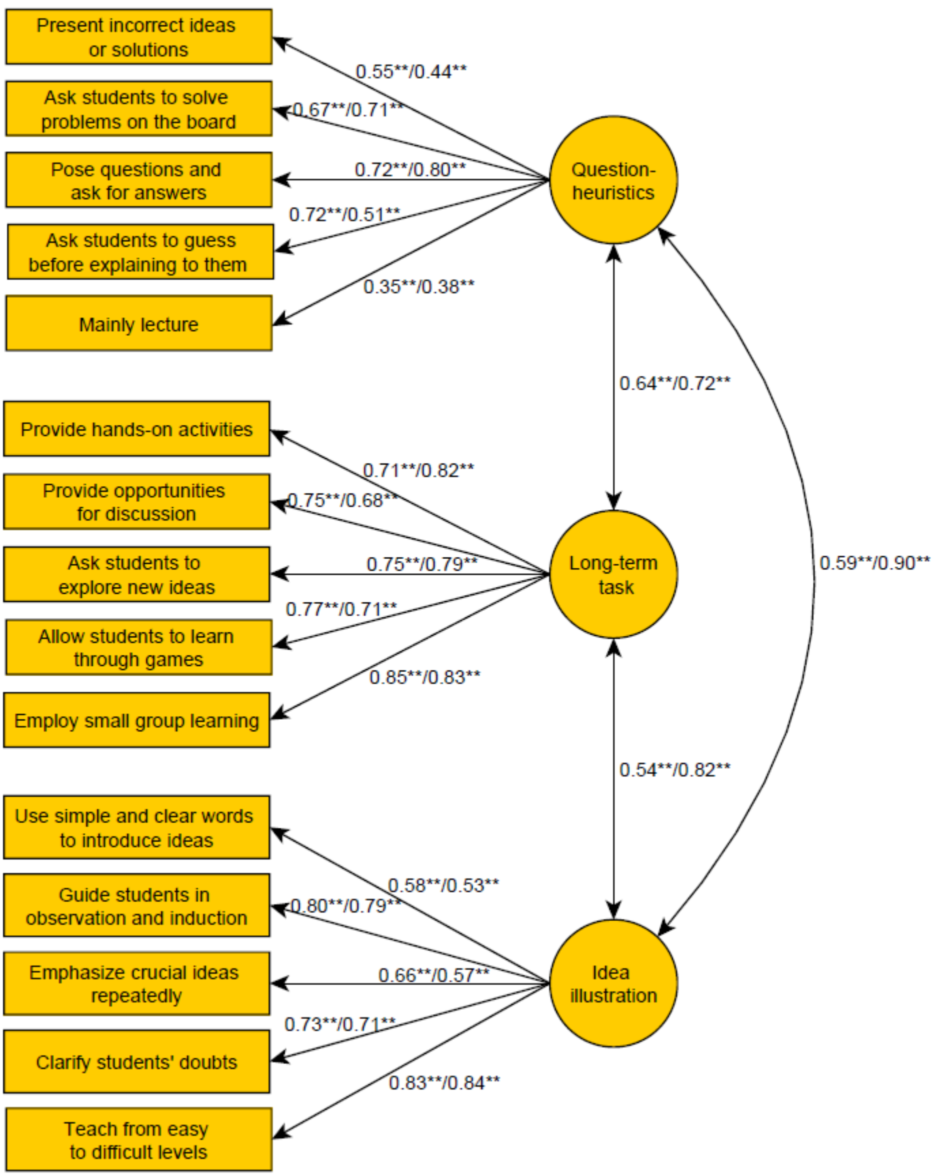

Figure 1. CFA models of the middle school level and primary school level (left values: middle school level; right values: primary school level)

Table 2. Differences of weighted average POCs between school levels and Cohen's ds

	Difference Primary-Middle	Cohen's d
Question-heuristics	0.10**	0.44
Long-term task	0.13**	0.50
Idea illustration	-0.02*	0.11

Note. A series of t tests was used to test the significance of the differences between weighted average POCs.
*$p < .05$. **$p < .01$.

At the same school level, the weighted average POCs between different factors were also significantly different (see Table 3); however, the Cohen's d effect sizes varied from no effect (d < 0.20) to a large effect (d > 0.80).

Table 3. Differences of weighted average POCs between factors and Cohen's ds

	Difference		Middle	Cohen's d		Middle
Factor	F1	F2	F3	F1	F2	F3
F1		0.05**	0.25**		0.18	1.15
F2	0.08**		0.20**	0.38		0.81
F3	0.13**	0.04**		0.66	0.22	
	Primary			Primary		

Note. F1, F2, and F3 represent *question-heuristics*, *long-term task*, and *idea illustration*, respectively. A series of *t* tests was used to assess the significances of the differences between weighted average POCs. Upper-right triangular values are for the middle school level and lower-left values are for the primary school level.
*$p < .05$. **$p < .01$.

The POC of individual indicators shows how crucial the indicator is for a great mathematics teacher. The results showed that (see Figure 3) all indicators except one (TD09) in *idea illustration* attained POCs higher than 90% for both school levels. The indicators in *long-term task* exhibited relatively high POCs from 82% to 92% at the primary school level and 68% to 82% at the middle school level. The indicators in *question-heuristics* had the largest range of POCs; the lowest POCs were 37% and 51%, and the highest were 85% and 92% for the middle and primary school levels, respectively.

Conclusion

This study was the first to collect nationwide Taiwanese students' perspectives on the teaching methods that a great primary or middle school mathematics teacher should focus on. Our results show that three distinct but correlated factors, *question-heuristics*, *long-term task*, and *idea illustration*, are crucial to being a great mathematics teacher in Taiwan for both school levels.

To be a great mathematics teacher, one must implement all of the indicators in the *idea illustration* factor, focusing on teaching from easy to difficult ideas, frequently repeating crucial ideas, and clarifying students' doubts. This factor includes teaching concepts shared from past to present in educational systems with a Confucian heritage (Hsieh, Lu, Hsieh, & Tang, in press). This factor may be perceived as a base factor that incorporates a teacher-led view. Our models for teaching methods suggest that a great mathematics teacher should not only adopt the suitable illustrative teaching methods in the base factor but also employ numerous other teaching methods embedding time-consuming investigation tasks (Brinker-Kent, 2000; Cai et al., 2009) and heuristic questioning that involves asking students questions and requiring them to guess or figure out others' solutions (Lin and Li, 2009; Li and Shimizu, 2009; Pang, 2009).

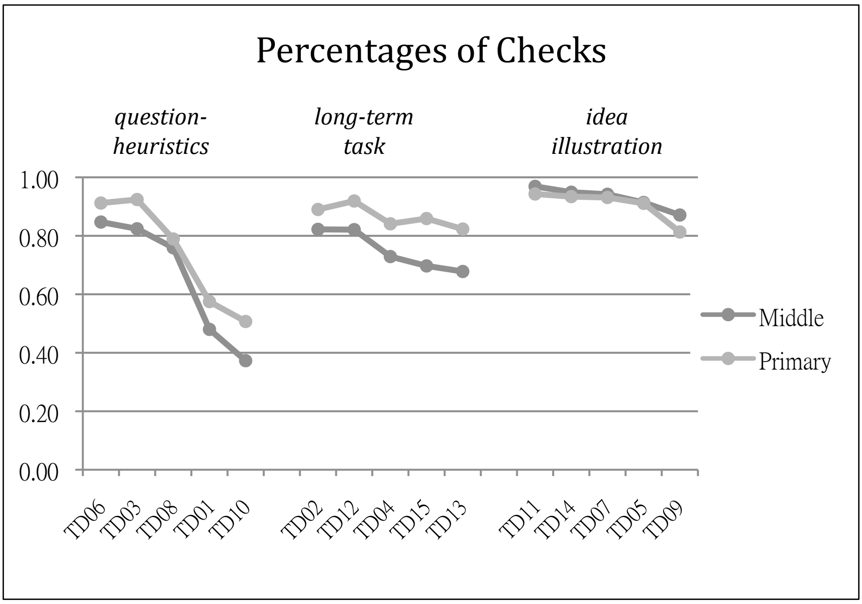

Figure 3. Percentage of checks for individual Indicators

Our data show that different school-level students rate the importance of the *question-heuristics* and *long-term task* factors differently. Primary school students consider these two factors significantly more critical than do middle school students for the criteria of being a great mathematics teacher. Different school-level students do not have different perspectives on the importance of the *idea illustration* factor.

The approach to investigating what constitutes a great mathematics teacher is only in its initial stage; additional studies should be conducted to explore the criteria in different dimensions of teaching competence at different school levels for different countries.

[1]They were identical except for a very few items that were not suitable to ask primary school students.

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