A preliminary study of Year 7 students' performance on algebraic concepts

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1. Background

This paper describes a preliminary study into the development of algebraic concepts among Year 7 students at one of the government schools (hereafter, School A) in Brunei Darussalam. Data were collected over a 2-week period, in April 2014. The purpose of the preliminary study was to obtain sufficient data with regards to how much algebraic knowledge and concepts were understood and remembered by Year 7 students from their previous year mathematics studies, particularly in solving algebra problems. The data collected will form a foundation towards the development of another larger study aimed at improving students' performance in algebra. The study involved 78 Year 7 students from School A, whose ages range from 11 to 14 years old and have completed and at least passed their Year 6 *Peniliaian Sekolah Rendah* (PSR) or the primary schools national examination. In the context of schooling in Brunei, the Year 6 primary students will sit for the PSR examination in order to transition from primary to secondary school. Year 7 marks the beginning of their secondary schooling.

The present study also aimed towards identifying which area of solving algebra problems students find most difficult or challenging to do, subsequently finding out the common error patterns and difficulties that have affected their performance. The students were administered a 25-item *Algebra Achievement Test*. Most of the questions on the test were taken from and related directly to the algebraic contents found in the Year 6 mathematics syllabus. It was expected that the data would provide a credible conclusion on the level of understanding and quality of the students' learning of algebraic concepts among the Year 7 students of School A.

2. Introduction

Solving algebra problems is one of the key components of the Brunei primary and secondary mathematics curriculum. The algebra topic is normally first taught in the primary level and then carried through to the secondary mathematics syllabus. In the curriculum and assessment guidelines from the Curriculum Development Department of Ministry of Education, the mathematics curriculum is designed to provide students with essential numeracy capabilities in the five strands of mathematics, namely numbers, algebra, measurement, geometry and statistics. These strands form an essential part not only in their personal livelihood and future work place, but in pursuing advanced mathematics and science courses that require a strong foundation of mathematics (Curriculum Development Department, 2011).

From past experience, students have had great difficulties when solving problems that involve calculations using algebraic concepts or manipulations. Nickson (2000) pointed out that students encounter difficulties when solving problems that involve manipulation of algebraic expressions and equations. There had been considerable studies on students' learning of mathematics and algebraic difficulties in the primary

and secondary schools in Brunei. Abdullah (1999) and Mohiddin (1998) have shown that many secondary students have difficulty coping with secondary school algebra. The data suggested that in algebra classes many lower secondary students merely learn to manipulate symbols, and cannot apply the learning in problem-solving situations. Later studies by Lim & Clements (2000a, 2000b) on Form 4 (equivalent to Year 10) 'O' Level students had shown that many secondary students were not learning to manipulate symbols correctly.

Vaiyavutjamai (2002) investigated 152 Form 3 students in two middle-secondary schools in Thailand who were administered a pencil-and-paper algebra test. Data showed that students experienced major difficulties because most of them were confused by the questions and did not know how to get the correct answer. In Fujii's (2003) investigation on the understanding of the concept of variable by 6th to 11th grade students in the United States and Japan showed that both samples had severe misconceptions of algebra. From this, it is imperative that an investigation is needed to identify and critically analyze the problems and misconceptions that students encounter at the early stages of algebra learning. According to Nickson (2000), algebra is a powerful problem-solving tool, therefore understanding algebra is central to students' ability to do mathematics. It follows that the teacher should enhance students' profound understanding and acquisition of algebraic concepts and thinking skills, in order to improve their performance in mathematics.

3. Methodology

3.1 Sample

In April 2014, the authors gained permission from three Year 7 classes, Years 7X, 7Y and 7Z, comprising 78 students in all, to be involved in the preliminary study. The classes were chosen for convenience -- proximity and they were the classes of one of the Year 7 teachers of School A. The other Year 7 classes apart from the ones mentioned in this study at School A did not participate in the preliminary study. All of the students participating in the preliminary study had studied algebra when they were in their primary schooling years particularly in Year 6. These 78 students generated performance data for the *Algebra Achievement Test*. Most of the Year 7 students in this preliminary study obtained grades of A, B, C or D in the Mathematics component of the PSR examinations in the previous year.

3.2 Instrument

The authors used the *Algebra Achievement Test* as the instrument in the preliminary study. This test was administered to the participating Year 7 students at School A.

<u>The Algebra Achievement Test</u>

Twenty-five questions comprise the test that was adapted from the study by Wessels (2009). Questions 1 to 15 were multiple-choice questions with four choices of answers namely a, b, c and d; and allocated one mark each. Questions 16 to 20 were word problems and allocated with three marks each. Questions 21 to 25 were solving equation problems, which were allocated two marks each. It was designed to test on five different areas of solving algebra problems and most of the questions were similar or related to the Year 6 mathematics syllabus. Given below are the aspects of the knowledge tested relating to the *Algebra Achievement Test*.

- Formulating algebraic expressions (5 questions)
- Mathematical language for algebraic expressions (5 questions)
- Manipulation of algebraic expressions (5 questions)
- Formulating algebraic equations from word problems (5 questions)
- Solving algebraic equations (5 questions)

Validity considerations

The Year 7 mathematics teachers agreed that all the questions on the test were valid – in the sense that the students might reasonably have been expected to answer the questions correctly as a result of their previous studies.

The validity of the *Algebra Achievement Test* was further checked by calculating Pearson product-moment correlation coefficients between students' scores on the *Algebra Achievement Test*, their PSR Mathematics grades and PSR English grades. The correlation coefficients, based on the 78 students in the three classes involved in the Preliminary study, are shown in Table 1.

Table 1. Pearson Product-Moment correlation coefficients between the Algebra Achievement Test scores and scores on the PSR Grades for Mathematics and English.

Subtest or Examination	Algebra Achievement Test	PSR Mathematics
Algebra Achievement Test	-	-
PSR Mathematics	0.761**	-
PSR English	0.546**	0.641**

** Correlation is significant at the 0.01 level (2-tailed).

Note: PSR Mathematics and English grades were regarded from 1(Pass) to 5(Grade A) and in this study; all students passed their PSR Mathematics and English.

Entries in Table 1 suggested that the *Algebra Achievement Test* measured many of the same attributes as the PSR Mathematics and English grades. From the data, there were positive correlations, significant at 0.01 level between the *Algebra Achievement Test* and the PSR Mathematics and English grades.

Test reliability

Based on the responses of the 78 students in the preliminary sample, the Cronbach Alpha reliability for the *Algebra Achievement Test* was calculated to be 0.761.

Test administration

The first author administered the *Algebra Achievement Test* to the three Year 7 classes. The test was administered one hour per class during normal lessons over a period of three days sometime in April 2014.

4. Preliminary study results and comments on the results

The lowest score gained on the *Algebra Achievement Test*, from among the 78 students, was 1 out of 40 (a student from Year 7Z), and the highest score, 30 out of 40 (a student from Year 7X). None of the participating students managed to gain full marks on the test.

Only one student (from Year 7X) responded correctly to question 18: "*Elsa is 6 cm taller than Anna. If the total height of the two girls is 350 cm, what is the height of each girl?*" i.e., Anna is 172 cm tall and Elsa is 178 cm tall. Further analysis of the students' responses to this particular question revealed that the students gave a wide range of different responses, e.g., 3 students responded with only either 172 cm or 178 cm but not both, which might indicate that these students did not fully understand the question. Most students (14 students) responded with 175 cm as their answer, followed by 11 students responding with 181 cm and 169 cm, 8 students gave 344 cm and 58 cm each as their answers.

Entries in Table 2 show the mean scores and the range of scores of the three classes. From the table, there is a variation in the class means, such that Year 7Z scored 6.2 while Year 7X scored 17.2. On average, students in Year 7X performed slightly better than the students in the other two classes. This may be expected as students in Years 7X and 7Y are categorised according to their scores in the Year 6 PSR examination.

Table 2. Descriptive statistics on the Algebra Achievement Test for the three classes.

Class (Year)	*No. of Students*	*Mean Score/40*	*SD*	*Lowest Score/40*	*Highest Score/40*
7X	29	17.2	5.9	5	30
7Y	24	16.2	4.95	7	25
7Z	25	6.2	3.3	1	16
Overall	n = 78	13.4	6.9	1	30

One of the questions in the *Algebra Achievement Test* was: "*A man is 3 times as old as his son. If their total age is 60 years, how old is the man?*" Entries in Table 3 show the number and percentage of correct and incorrect responses to this question in each class, and indicate the overall performance on the question, together with the extent of between class-differences.

Table 3. Data on correct and incorrect responses to the question (on the Algebra Achievement Test).

Class (Year)	*No. of Students*	*Correct*	*Number (and %) of Incorrect Responses*		
		45	*180*	*20*	*Other Errors*
7X	29	4 (13.8%)	0 (0.0%)	5 (17.2%)	20 (69.0%)
7Y	24	1 (4.2%)	1 (4.2%)	10 (41.7%)	12 (50.0%)
7Z	25	0 (0.0%)	9 (36.0%)	4 (16.0%)	12 (48.0%)
Overall	n = 78	5 (6.4%)	10 (12.8%)	19 (24.4%)	44 (56.4%)

Four students from Year 7X, one student from Year 7Y and no student from Year 7Z gave the correct answer for the question, while the remaining 73 students gave incorrect responses. The most incorrect response the students gave was 20 obtained by dividing 60 by 3, followed by 180 most likely from multiplying 60 by 3.

Table 4 below provides additional data on selected questions from the *Algebra Achievement Test.* The questions listed in the table are chosen based on the least percentage of correct responses and the most common error responses students gave.

Table 4. Year 7 students' performance on selected questions on the Algebra Achievement Test.

Question	% Correct	Most Common Errors
9. Take away **7** from the product of ***m*** and ***n+1***. (MCQ)	26.9%	Most students chose the answer $7 - m(n + 1)$ with 30 students.
13. Express as a single fraction: $\frac{3m}{4} + \frac{m}{3}$ (MCQ)	10.3%	The most common error response is $\frac{4m}{7}$ where students just added the terms in the numerators and added the terms in the denominators. Only 8 students managed to answer this question correctly.
14. Simplify the expression: $\frac{8a}{3b} \div \frac{6a}{5b}$ (MCQ)	25.6%	24 students gave $\frac{48}{15}$ as their answer.
23. Solve for *x*: $10 - x = 8$	11.5%	33 students gave the answer 18 as the incorrect response. They added 10 to the right hand side and forgotten about the minus '–' sign of the variable *x*.
24. Solve for *x*: $32 - 3x = 18$	7.7%	25 students gave the answer as $\frac{50}{3}$ or 16.66 as their incorrect response. Similar method as question 23 above was employed by the students. They added 32 to the right hand side and forgotten to divide by '–3' to solve for the variable x.

5. Discussion of results of the *Algebra Achievement Test* and implications

The overall results of the three classes on the *Algebra Achievement Test* indicated that very few students were able to demonstrate a solid grasp of the basic algebraic knowledge content covered by the test questions. Only one student from Year 7X managed to score the highest mark of 30 out of a total 40. From the authors' perspective, the levels of performance on solving algebraic problems were much less than might have been expected of the three classes, composed of students who had passed (at least) their PSR Mathematics national examinations.

The quantitative analyses done on the students' results suggested that most students have difficulty in formulating equations when solving word problems with only 23.6% of the students managing to give the correct answers, followed by a percentage of 25.9% on questions that required them to manipulate algebraic expressions correctly. However, students fared better when solving algebraic equations that are given although some students still demonstrate weak arithmetic skills and made errors of an

arithmetical nature causing them to make algebraic errors as well. Students with low performance tended to make errors mainly because of a poor understanding of negative numbers, a poor understanding of what an equation means and weaknesses in basic arithmetic.

Table 5. Distribution of students' percentage correct responses in the Algebra Achievement Test.

Attribute	*Item numbers*	*Percentage of correct responses*
A. Ability to form algebraic expressions	1-5	39.7%
B. Proficiency of mathematical language used in real-world problems	6-10	52.6%
C. Ability to manipulate algebraic expressions correctly	11-15	25.9%
D. Ability to formulate word problems into algebra equations and consequently solve the equations	16-20	**23.6%**
E. Ability to solve given algebra equations	21-25	26.4%

Although the students in Year 7X had a higher mean score than the other two classes in the *Algebra Achievement Test*, the overall results could hardly be classified as "good" in this particular test. Overall, the 78 Year 7 students did poorly on the *Algebra Achievement Test* even though the questions were based on the Year 6 syllabuses. These students had been taught basic algebraic skills from Year 4 to Year 6 in their primary schooling, yet results indicate clearly that they did not remember much of what they had been taught in previous years. Indeed, the overall mean score of 13.4, out of a possible 40, indicated that the Year 7 students managed to answer correctly only about 34% of the questions on the *Algebra Achievement Test*.

6. Conclusion

The preliminary study was expected to identify the area in solving algebra problems that students find most difficult or challenging to do and subsequently finding out the common error patterns and difficulties that have affected their performance. The findings from this preliminary study pointed towards the conclusion that many of the Year 7 mathematics students in the study's sample still demonstrated a poor grasp of basic algebraic knowledge and skills. The students still experience difficulties when solving problems that involve manipulation of algebraic expressions and equations. Likewise, these students could not apply their learning to problem solving situations such as formulating equations from word problems. The results on the *Algebra Achievement Test* raise the question on the extent to which students are able to cope with the more difficult and complex topics in their future O-level mathematics syllabus. Students with low test performance tended to make errors mainly because of a poor understanding of negative numbers, a poor understanding of what an equation means and weaknesses in basic arithmetic. Therefore, from the analyses of the preliminary study data there is the need of an in-depth study into the development of effective teaching strategies in the teaching and learning of algebraic concepts and skills to be conducted further in schools in Brunei Darussalam.

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