Developing the number knowledge of low-attaining first- to fourth-graders

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Ten-structured thinking is one of the major conceptual advancements that children need to develop in the early elementary grades (Fuson et al., 1997). However, traditional mathematics instruction in the early elementary grades often does not focus on developing the ability to organize number according to its decimal structure (Ellemor-Collins & Wright, 2011). This paper describes one phase of a study aimed to develop the ten-structured thinking of Filipino first- to fourth-graders who were identified by their teachers as low-attaining in mathematics. To provide a context, the place of ten-structured thinking in the Philippine mathematics curriculum will be discussed in relation to the scholarly literature.

Literature review

The ability to perform the four arithmetic operations on multi-digit numbers is an important milestone in the primary mathematics classroom. However, this goal is not easily achieved. Children may either apply incorrect procedures or forget learned algorithms when they perform arithmetic tasks (Hiebert, 1984). The consensus was that children make mistakes because they do not understand the reason behind the standard (column) algorithms. As such, in various curricula, including that in the Philippines, the teaching of the standard algorithm follows lessons on place value. Visual or concrete materials (e.g., base-ten blocks) are also encouraged.

Place value instruction

Traditional place value teaching is seen as a preparation for learning the standard algorithms (Ellemor-Collins & Wright, 2011). Some related competencies in the Philippine curriculum for Grade 1 are "visualizes and gives the place value and value of a digit in one- and two-digit numbers," and "renames numbers into tens and ones." (DepEd, 2012a, p. 11). These competencies are covered in one page in the official textbook (DepEd, 2012b, p. 106). In the lesson, 47 stars are arranged in 4 rows of 10 and 1 row of 7. It then states that 47 is composed of 4 tens and 7 ones. Next, the text mentions that in 47, the value of 4 is 4 tens or 40, and the value of 7 is 7 ones or 7. A place value chart is presented at the end of the lesson. The next three pages consist of exercises, mostly requiring students to give the number of tens and ones in a given number (or vice versa).

Using Thompson's (2000) terminology, the lessons and exercises described above focus on knowing a number's *column value*—or knowing that 47 is 4 in the tens column and 7 in the ones column. However, Ellemor and Wright (2011) assert that such knowledge is limited in terms of developing ten-structured thinking. For example, it does not help a child see that 47 and 10 more is 57. It also does not facilitate solutions based on the decimal structure (e.g., 34 + 19 is 34 + 20 and 1 less). Thus, Ellemor and Wright (2011) suggest a need to emphasize Thompson's (2000) notion of *quantity value* in place value instruction. They extend the notion of quantity value to mean strengthening the network of number relationships (e.g., 47 is 40 and 7; it is also 10 more than 37, 10 less than 57, 3 less than 50, and so on).

The use of base-ten supports

A second feature of place value instruction is the use of base-ten supports. For example, in the official Grade 1 textbook (DepEd, 2012b), the lessons on addition with or without regrouping (p. 82 and p. 85) use base-ten blocks as illustrations of corresponding addition tasks.

It appears, though, that representing how quantities may be composed or decomposed into ones, tens or hundreds is not sufficient to develop understanding. As Fuson et al. (1997, p. 132) argue, children are not "instamatic" cameras—they are not guaranteed to interiorize the ideas that a ten-structured object intends to support. Rather, what a child sees when looking at an object is largely determined by the mental representations that are already available in the child's mind.

Consider a child who is shown five strips of ten squares, and three more squares (53 squares). Suppose the child counts the squares on the strips this way: 10, 20, 30, 40, 50. However, to count the three additional squares, the child continues: 60, 70, 80. This example demonstrates how an adult and a child may see objects differently. According to Cobb and Wheatley (1988), when a child counts strips as 10, 20, 30 it does not imply that the child sees 30 squares. Instead, the child may just have learned a new way of counting. The child may just as well have counted the strips as A, B, C. The mental representations currently available to the child do not allow him or her to construct ten as a unit. Thus, each strip may be seen as ten ones, or as one ten, but not simultaneously.

Theoretical Framework

Gray and Tall (1994) introduce the notion of a *procept*, or a mental amalgam of process and concept. Because a mathematical symbol may represent both a process and a concept the difference between high- and low-attainers are in how they "see" the symbol. For example, the symbol "9" may represent the process of counting 9, or the concept of "nine-ness". A child may also realize that a mathematical object (such as 9) can be represented using different symbols. Thus, various other representations for 9, such as 1+8, 2+7, 10–1, etc, may also be available to the child. As such, the nature of a procept is dependent on the internal network of relationships in the child's mind.

This theory can explain why some people find mathematics really easy while some struggle. Gray and Tall (1994) talk about the *proceptual divide*. Low-attaining children rely primarily on procedures and try to be good at them, whereas high-attaining children can view symbols as objects and work with them in flexible ways. For example, the problem "9+4" can be solved using different levels of sophistication. A child can solve the problem by counting. This child sees the symbol "9+4" as a procedure. By contrast, a more advanced child can draw from a range of number relationships. Since 9 is also 10–1, then 9+4 can be seen as 10+3, which is 13.

This framework implies that success may be achieved by broadening the relevant relationships in the child's mind. In the case of arithmetic, ten-structured thinking is an important anchor to which number relationships may be tied (Ellemor-Collins & Wright, 2011).

Guided by the above theoretical perspective, the aim of this paper is to investigate ten-structured thinking among a group of Filipino students. Specifically, two research questions are addressed:
(1) To what extent do low-attaining Filipino children demonstrate ten-structured thinking when solving number tasks?
(2) Can their thinking be developed through a short-term small-group intervention program carried out by non-professional university students?

Method

The study involves 108 first- to fourth-graders in a Philippine public school who were identified as low-attaining by their teachers. The school services a large number of economically disadvantaged families near the area. The study was carried out within the National Service Training Program (NSTP), which is a one-year requirement for all Filipino tertiary-level students. The tutors involved in this study are sophomore mathematics majors who are taking their NSTP.

The NSTP tutors underwent two skills training sessions of four hours each. The researcher handled the training. During the training, the tutors were introduced to ten-structured thinking. The individual assessment and sample lessons were also discussed. All tutors were provided with an assessment kit, the learning modules, and worksheets.

The learning tutorials were carried out over two hours each Saturday, for 16 Saturdays. Each tutor was assigned to teach one to three children for the duration of the program. Attendance on the part of the children was not required. It was a challenge to maintain constant attendance, especially when the weather was bad, or when there was no parent to bring the child to school on a Saturday afternoon. Because of the sporadic attendance, only 19 of the 108 children were assessed post-intervention. However, during each session, there were at least 40 children, and some children were present during all sessions.

Assessments

The pre- and post-assessments were carried out individually using the child's first language, Filipino. The tasks were informed by previous work on ten-structured thinking and place value. These included subitizing, incrementing/decrementing by 10s or 100s, and representing number using groups of 10. The goal was to assess various number relationships that are available to the child. For each task, the tutors were provided a list of solution strategies from which they could select the strategy that best describes what they observed. The list of solution strategies was based on the literature and on pilot work with a different group of sophomore university students.

Instruction

The tutors used three modules (with accompanying worksheets) that were prepared by the researcher. The modules were (1) structuring numbers 1-20, (2) sequencing numbers 1 to 100, and (3) structuring numbers. Module 1 developed facility with adding and subtracting numbers in the range 1 to 20, with a focus on helping children use 10 as a unit. The main instructional material was the tens frame. Module 2 developed the ability to increment and decrement by 10s with the hundreds chart or blank hundreds chart as an initial guide. Module 3 focused on representing numbers in tens and ones (e.g., 48 = 4 tens and 8 ones) as well as addition and subtraction. The main teaching strategy involved tasks that require children to shade numbers on a rectangular grid. Modules 2 and 3 have provisions for extension to numbers through 999.

Results

Pre-assessment results

Subitizing (Wright et al., 2012)

Children were first shown a tens frame card with ten dots, to help establish the fact that there were ten dots on a card. Next, various cards showing a number of dots were flashed for around two seconds. Children were asked to identify the number shown on the card. Almost a third of the children succeeded in identifying 7 from a ten frame (Figure 1), but less than half succeeded in the larger number task (Figure 2).

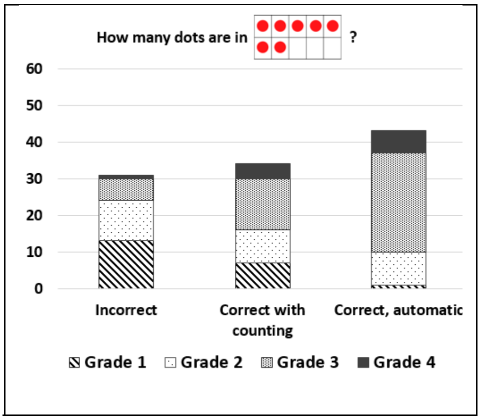

Figure 1. Subitizing task (7)

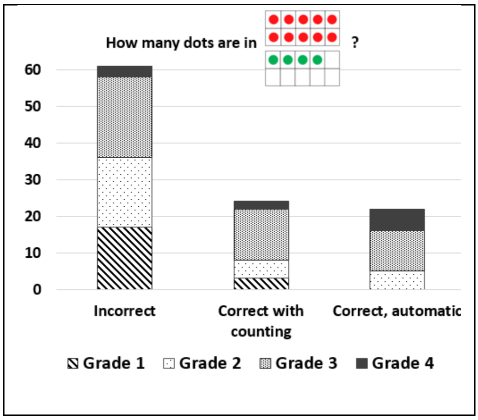

Figure 2. Subitizing task (14)

Base-ten material task (Berman, 2011)

Children were shown a tens strip, to help establish the fact that there were ten dots on a strip. Next, they were shown 25 dots and 7 tens strips (Figure 3), and were asked to identify 52 dots. Only about 15% of the children selected 5 strips and 2 ones on their first attempt (see Figure 4).

Grid task (Tomazos, 2011)

The tutor first demonstrated how 23 squares could be enclosed in a grid (Figure 5). Children were then asked to draw around 48 squares. Less than 29% of the children applied a grouping strategy (Figure 6).

Star task (Berman, 2011)

A 7 × 5 array of 35 stars is shown, and children were asked to write down the number (35) of stars in the box. The tutor encircled the digit 5, and said, “Encircle the stars that are represented by this.” Next, the tutor encircled the digit 3, and repeated the instruction. While more than half identified 5 stars correctly, only about 10% correctly identified what the digit 3 represents (Figure 7).

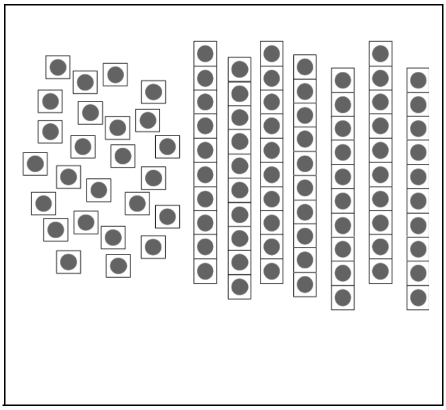

Figure 3. Base-ten materials task

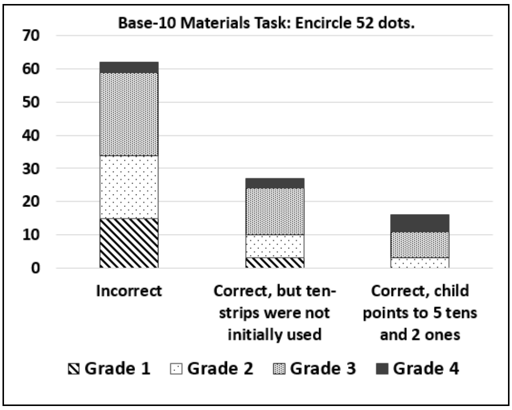

Figure 4. Base-ten materials task results

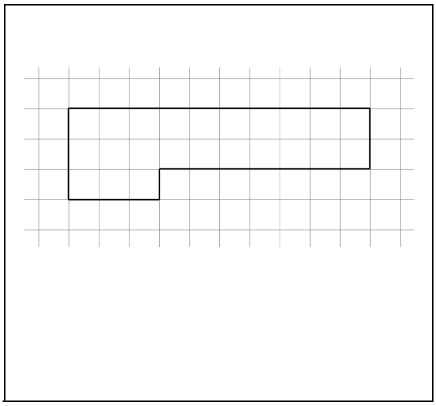

Figure 5. Grid task

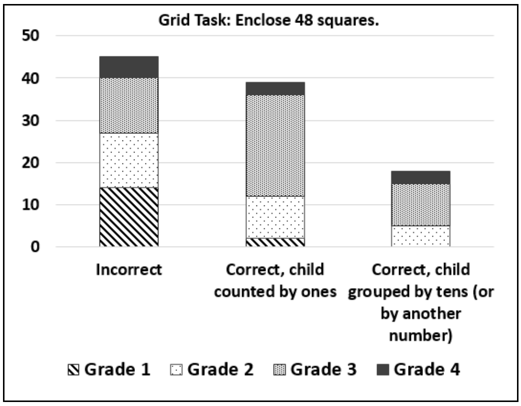

Figure 6. Grid task results

Decrementing task (Wright et al., 2012)

The expression 67 – 10 was printed on a card and shown to the child. Close to half were incorrect, and less than 10% of the children solved the problem automatically.

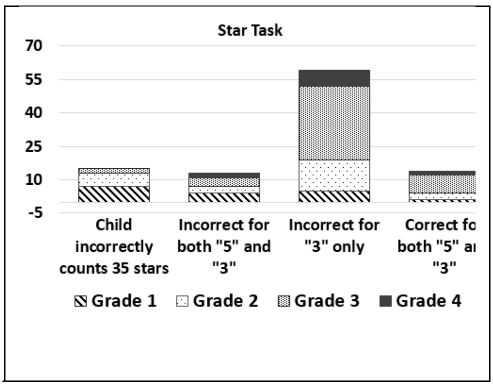

Figure 7. Star task results

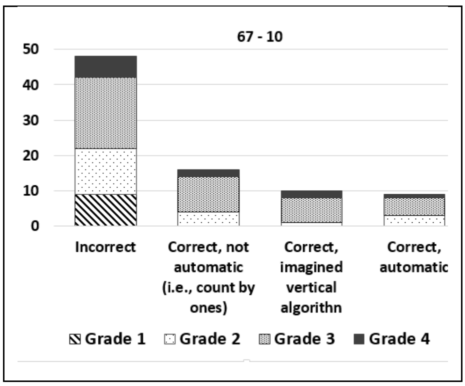

Figure 8. Decrementing task results

Traditional place value task
In this task, the number 352 was printed on a card, and children were asked to identify the digit in the tens place. Children were relatively more successful in this task (Figure 9) than they were in any of the previous tasks.

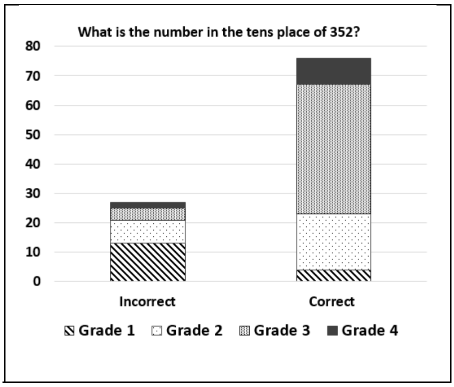

Figure 9. Traditional place value task results

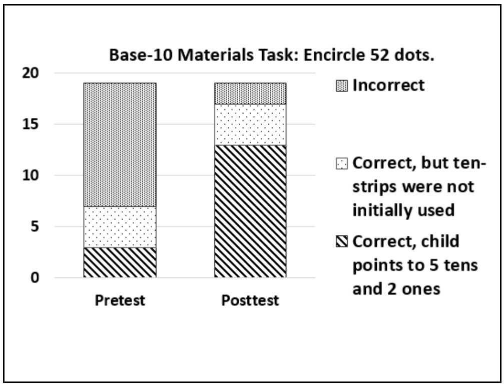

Figure 10. Base-ten materials task (pre- and post-intervention)

Post-intervention data
The intervention is analyzed through the progress of the 19 children who were assessed before and after the intervention. The results for the Base-ten materials task, Grid task, Star Task and Decrementing task are shown on Figures 10, 11, 12, and 13, respectively.

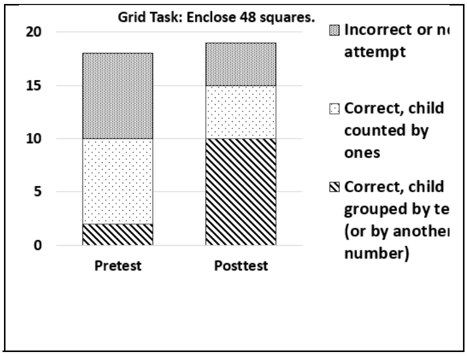

Figure 11. Grid task results

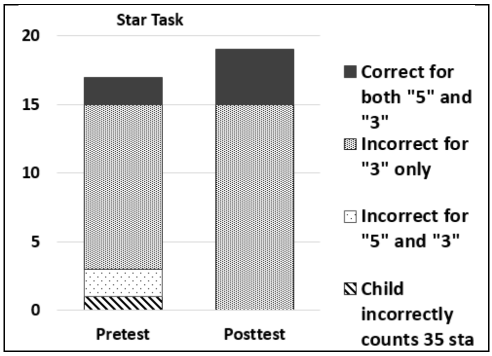

Figure 12. Star task results

Children's solutions were more advanced after the intervention. In the posttest, more children were able to represent numbers by using groups of tens, and more children provided automated answers to the decrementing task. Still, there were children who continued to rely on unitary counting strategies. For these children, it seems that the 16 weekly sessions over the course of one school year was not sufficient for developing ten-structured thinking.

Conclusion
Although this study involved one school, the results provide a direction for classroom teaching as well as for future work in the area. A wide range of number relationships

anchored on ten-structured thinking has been identified by researchers as crucial in mathematical development, yet it is striking that low-attainers may not have interiorized these relationships. They were most successful in the traditional place value task, where they were asked to identify the tens digit of a number. By contrast, they performed very poorly on the Star Task. These results indicate that knowing that 5 is the tens digit of 352 is not the same as knowing that the 5 represents 5 tens or 50. Using Thompson's (2000) terminology, the children can understand a number's column value, but not its quantity value.

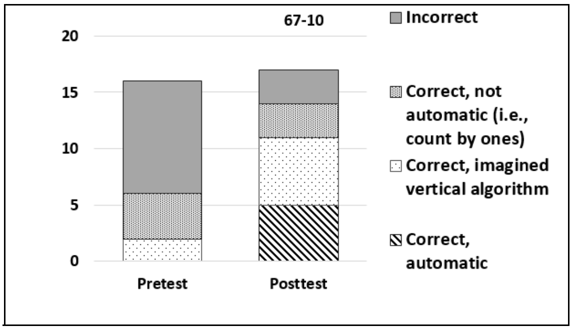

Figure 13. Decrementing task results

At a basic level, only a few children can represent a number using tens and ones, as shown in the Base-ten materials and Grid tasks. While visuals are present in the official Grade 1 textbook (DepEd, 2012b), these are pre-constructed—children simply need to write down the correct number represented by the diagram. A teaching implication is to provide more opportunities for children to construct the representation themselves. Further, the use of pre-constructed representations must be questioned in light of this study's results. The large number of children who failed in the Base-ten materials or the Grid task are not likely to see the pre-constructed representations as tens and ones when their current cognitive structures do not allow them to see ten as a unit in the first place (Cobb & Wheatley, 1988; Fuson et al., 1997)

The curriculum (DepEd, 2012a) and official Grade 1 textbook (DepEd, 2012b) also include counting by tens among the learning competencies. However, the results indicate that low-attaining children may just have learned a new way of "counting." Based on Cobb and Wheatley's (1988) analysis, the children in this study may just be uttering the numbers without forming connections with a number's quantity. For example, they cannot use their count up procedure to answer a simple decrementing task such as 67 – 10. In fact, some children attempted to use the learned vertical algorithm to solve a task that could have been solved mentally. As Ellemor-Collins and Wright (2011) suggest, children may need more opportunities to count up *and* down by tens *off* the decade (for example, 3, 13, 23, 33,…).

Another outcome from the study is the children's lack of familiarity with teen numbers. Results from the subitizing task indicate that majority of low-attainers, including a few fourth-graders, could not identify 14 as 10+4. This result, together with the limited cognitive network of number relationships available suggests that these low-attainers may continue to rely on inefficient procedures. As Gray and Tall (1994) argue, these low-attainers need to move on to the next stage of encapsulating the procedures as part of a procept. Otherwise, by the time they have mastered a procedure, the lessons have already moved on, and they are perpetually left behind.

The pedagogical implications have been tried out during the intervention phase of the study. The results are quite promising, given that the schedule was not ideal (e.g., attendance is not compulsory; few number of sessions). Still, the relative lack of success in the Star Task post-intervention suggests that ten-structured thinking may develop over a prolonged period of time, and cannot be taken for granted.

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