Comparing teacher perceived students' errors and students' errors: Match or mismatch?

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Introduction

Radatz (1980) claimed that failure in concept mastery might cause students' errors. He also explained that these errors are usually systematic and can be analysed, and defined this method of analysis as "error techniques." His idea found support in Lannin, Barker and Townsend (2007) as well as in Borasi (1987). They all agreed that students' error patterns could be used to identify students' learning difficulties and misconceptions in mathematics. In order to understand students' difficulties and misconceptions in mathematics, teachers need to perceive students' errors. However, teacher perceived students' errors (TPSE) might not be the actual students' errors (SE). Hence, comparisons between TPSE and SE are important because a match or mismatch could influence their teaching and students' learning.

This study focused on the topic *Time* because primary school teachers often complained that their students have difficulties in solving problems related to conversion of units and calculation of duration. In addition, researchers have concluded that time is a complex concept and is not easy to teach to children (Burny, Valcke & Desoete, 2009).

Error analysis aims to identify the types of errors and to plan a remediation course. The advantage of error analysis is that analysis of students' ways of thinking in mathematics can actually help teachers reflect on their teaching and improve their teaching strategies (Doerr, 2006). But, most error analysis studies focused on identifying students' errors and seldom matched students' errors that teachers perceived with the actual errors students made. Therefore, there is a need to investigate the match or mismatch between TPSE and SE.

Objective

This main objective of the project is to develop a two-pronged Cognitive Diagnostic Assessment Model (CDAM) for primary school learning of time. This paper reports the first stage of the project, which aimed to determine the extent that TPSE match with their SE in solving time-related problems involving conversion of units and duration.

Methodology

The first stage of the project involved 2 mathematics teachers and their 10 students from each of the three types of primary schools in Malaysia, namely the National School (*SK*), National Type Chinese School (*SJKC*) and National Type Tamil School (*SJKT*). The sample consisted of 6 mathematics teachers and 30 students. They were expert teachers who had taught primary school mathematics for more than ten years. They randomly selected 10 average-performing students in mathematics to participate in the study.

The data were collected using a diagnostic test comprising 11 items, which were selected from the topic of time in the Mathematics Paper of the Primary School Assessment Tests from the year 2009 to 2013. The test is a public examination for primary students who have completed six years of primary education.

The researchers selected 42 items for the panel of 6 expert teachers to identify the errors that their students might make in answering the items and these errors are known as TPSE. Only five expert teachers managed to complete the tasks and reached a consensus on 5 items. Since the number of items was insufficient, 6 more items, which were agreed by at least three out of the 5 expert teachers, were also selected. Hence, a total of 11 items were selected to form the diagnostic test. The test was then administered to 30 students. They were asked to answer all the items and show all their works.

All the 30 students' responses and the 5 teachers' responses were compiled and analysed. First, the teachers' responses were collected and sorted according to the similarities and differences of the TPSE. Next, their students' responses were analysed to identify SE. Finally, comparisons between the TPSE and SE were then carried out based on the type of schools. The TPSE are considered as matching the SE if the students committed the TPSE.

Results and discussion

The results are presented and discussed according to the type of schools in the following sections. Table 1 shows the percentage of match and mismatch between the TPSE and SE for the *SK*.

As shown in Table 1, there are seven items that have higher percentages of match than mismatch, namely Items 1, 3, 6, 7, 9, 10 and 11. Out of the seven items, there are five (Items 3, 6, 7, 9 and 10), which have zero percentage of mismatches. For Items 1 and 11, 70% of the students committed the errors as perceived by their teachers, but 20% and 10% of the students committed the errors that were not perceived by their teachers, respectively. The high number of items that shows a higher percentage of matches than mismatch indicates that the *SK* teachers understood the difficulties faced by their students in answering the items quite well.

However, for Item 8 none of the students committed the error as perceived by their teachers. In fact, 80% of the students committed an error that was not perceived by their teachers. Item 8 required students to find the duration given the starting date and the ending date. The *SK* teachers perceived that their students might not be sure whether the two given dates in the item should be included in finding the duration. This uncertainty arose because in this type of problem, students often calculated the duration between the starting date and the ending date. While the analysis of the data showed that the students from the *SK* were able to determine that the two given dates are inclusive, they had problems in identifying the number of days of the different months of the year. This error was not perceived by their teachers resulting in a high percentage of mismatch.

In addition, both Items 4 and 5 show zero percentage of match and mismatch. The result indicates that none of the students committed the errors as perceived by their

teachers, and also none of the students committed the errors that were not perceived by their teachers. In other words, all of the students gave correct responses for these two items. These items did not seem difficult to them. In fact, the students did better than what their teachers had expected.

Table 1. Percentage of match and mismatch for the SK

Item	Content Area	No. of students who committed the TPSE	% of match	No. of students who committed errors that were not the TPSE	% of mismatch
1	The concept of "one week after"	7	70	2	20
2	Subtraction of two durations	1	10	2	20
3	Concept of average involving time	2	20	0	0
4	The concept of "a quarter hour"	0	0	0	0
5	Addition involving time	0	0	0	0
6	PS: Find the end time of an event	3	30	0	0
7	PS: Find the start time of an event	4	40	0	0
8	PS: Find duration from a given date to another date	0	0	8	80
9	Relationship between units of time	4	40	0	0
10	Division involving time	6	60	0	0
11	PS: Find end time of an event involving day and hour	7	70	1	10

Note: PS = Problem Solving

Table 2 shows the percentage of match and mismatch between the TPSE and SE for the *SJKC*. As shown in Table 2, there are six items (Items 3, 4, 6, 7, 9 and 10) that have higher percentages of match than mismatch. Four out of the six items (Items 3, 4, 9 and 10) show zero percentage of mismatch. For Items 6 and 7, 50% and 60% of the students committed the errors as perceived by their teachers, but 20% of the students committed the errors that were not perceived by their teachers for both the items. The high number of items with higher percentages of match than mismatch, indicates that the *SJKC* teachers also understand quite well the difficulties faced by their students in answering the items.

Items 1, 8 and 11 show higher percentages of mismatch than match. Item 11 shows the biggest difference (60%) between the percentage of match and mismatch. This item involved addition of time that exceeds 24 hours (more than 1 day). The teachers perceived that their students might forget or get confused about the time. Yet, only two students committed this type of error. The rest of the students did not commit this error. Some of them did not understand the requirement of the item while others did not understand the relationship between the start time, end time and duration. These are the errors that were not perceived by the teachers resulting in a high percentage of mismatch.

Both Items 2 and 5 show zero percentage of match and mismatch. The result indicates that none of the students committed the errors as perceived by their teachers,

and also none of the students committed the errors that were not perceived by their teachers. Item 2 involved subtraction of time. The teachers perceived that their students might have problems because the solution involved regrouping of hours to minutes. However, none of the students committed the error. This indicates that the item might be easy for all students. Similarly for Item 5, the teachers perceived that their students might be unable to do the conversion of the units. Yet, none of the students gave incorrect response to this item. In other words, all of the students gave correct responses for these two items. These items seemed to be not difficult for them. In fact, they were better than what their teachers had perceived.

Table 2. Percentage of match and mismatch for the SJKC

Item	Content Area	No. of students who committed the TPSE	% of match	No. of students who committed errors that were not the TPSE	% of mismatch
1	The concept of "one week after"	3	30	5	50
2	Subtraction of two durations	0	0	0	0
3	Concept of average involving time	3	30	0	0
4	The concept of "a quarter hour"	9	90	0	0
5	Addition involving time	0	0	0	0
6	PS: Find the end time of an event	5	50	2	20
7	PS: Find the start time of an event	6	60	2	20
8	PS: Find duration from a given date to another date	2	20	4	40
9	Relationship between units of time	7	70	0	0
10	Division involving time	4	40	0	0
11	PS: Find end time of an event involving day and hour	2	20	8	80

Note: PS = Problem Solving

Table 3 shows the percentage of match and mismatch between the TPSE and SE for the *SJKT*. As shown in Table 3, there are only three items (Items 3, 9 and 11) that have higher percentages of match than mismatch. But all of the three items show zero percentage of mismatch. In addition, for Item 11 none of the *SJKT* students answered correctly and all of them committed the error as perceived by their teacher. The low number of items with higher percentages of match than mismatch, indicates that the *SJKT* teacher did not understand quite well the difficulties faced by his students in answering the items.

However, there are three items (Items 2, 5 and 6) that show higher percentages of mismatch than match. Item 2 involved the subtraction of two durations. The teacher perceived that his students might be unable to convert the unit from hour to minutes as the subtraction involved regrouping between these two units. However, the students did not commit the error as perceived by their teacher. Instead some of them performed the wrong algorithm for the operation while others made careless mistakes. For Items 5 and 6, the percentage of mismatch is 30% whereas the percentage of match is 10% and 20%,

respectively. For Item 5, the teacher did not expect that some of his students performed subtraction instead of addition. For Item 6, most of the students could not understand the requirement of the item. They calculated the duration instead of the arrival time. These were the errors that were not perceived by the teacher.

In contrast to the *SK* and *SJKC* teachers, the *SJKT* teacher did not perceive any students' errors for Items 1, 4, 7, 8 and 10 as indicated by the letter N in Table 3. In other words, the students were expected by their teacher to be able to answer the five items correctly. However, the result showed otherwise. Only one student was able to answer Item 1 correctly. The other nine students made the mistake of assuming that April has 31, 29 or even 24 days. Only four out of ten students gave the correct response to Item 7. Many of them did not understand what was required of the item and thus chose an inappropriate operation to answer it. Item 8 was about the calculation of duration from a given date to another date. The result shows that only one out of ten students answered the item correctly, three students did not answer the item at all and six of the students gave incorrect responses. These students were confused about the inclusiveness of the two given dates in calculating the duration. The *SJKT* teacher perceived that Item 10 would not be a difficult question for his students as it only involved the division of time, which required students to divide a given duration by a constant. The students had to convert the remaining hour into minutes in order to answer this item correctly. Unexpectedly, seven out of 10 students were unable to answer this item because they did not convert the remaining hour into minutes correctly.

Table 3. Percentage of match and mismatch for the SJKT

Item	Content Area	No. of students who committed the TPSE	% of match	No. of students who committed errors that were not the TPSE	% of mismatch
1	The concept of "one week after"	N	-	9	90
2	Subtraction of two durations	0	0	3	30
3	Concept of average involving time	8	80	0	0
4	The concept of "a quarter hour"	N	-	4	40
5	Addition involving time	1	10	3	30
6	PS: Find the end time of an event	2	20	3	30
7	PS: Find the start time of an event	N	-	6	60
8	PS: Find duration from a given date to another date	N	-	6	60
9	Relationship between units of time	3	30	0	0
10	Division involving time	N	-	7	70
11	PS: Find end time of an event involving day and hour	10	100	0	0

Note: PS = Problem Solving; N = The teacher did not perceive any students' errors

Conclusion

Among the three types of schools, the *SK* has the highest number of items (7 items) having higher percentage of match than mismatch between TPSE and SE. This is followed by the *SJKC* (6 items) and *SJKT* (3 items). In contrast to the *SK* and *SJKC* teachers, the *SJKT* teacher did not perceive any students' errors for 5 items. In other words, the teacher perceived that his students would be able to answer the five items correctly. But, the result shows otherwise.

An interesting result of this study is that there are only a few items in which the students did not commit the TPSE. There are three items for the *SK* (Items 4, 5 and 8), two items for the *SJKC* (Items 2 and 5) and only one item for the *SJKT* (Item 2). This implies that generally the teachers in the three types of schools were able to identify their students' weaknesses in the topic of time. According to Curwin (2014), teachers who are able to identify their students' possible errors and then make use of them in the teaching and learning process, such as using these errors as counter examples, or teaching and learning through known mistakes can enhance students' learning of mathematics.

However, there are also some errors committed by the students which were not perceived by their teachers because the students did some careless mistakes and the teachers might predict the common errors which their students would likely to commit based on their experience of teaching time. This shows that teachers may need to take note of these errors in future which in turn will help them understand better their students' difficulties in learning time.

The result also indicates that there are two types of mismatch between the TPSE and SE. In the first type, the teacher perceived certain types of students' errors but their students did not commit the errors. In the second type, the teacher did not perceive certain types of students' errors but their students committed the errors.

In conclusion, the results of this study may help teachers gain a better insight into their students' difficulties in solving problems involving time. Teachers can then focus on the common types of errors that students committed and subsequently diagnose the possible causes so that they can help their students learn time more effectively.

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