Using a PDCA cycle to develop teaching proficiency in prospective mathematics teachers
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Introduction
The Plan-Do-Check-Act (PDCA) Cycle, a collaborative professional development approach that is also referred to as the Deming Cycle, is a popular tool often used in the teaching improvement process (Sallis, 2005). The origins of the PDCA Cycle can be traced to Walter Shewhart who invented the concept of control charts and later expanded his work to include publications in science and statistical inference. After reading Shewhart’s work, W. Edward Deming began to champion Shewhart’s insights and ideas. Deming further developed Shewhart’s methodologies into what is now known as the PDCA Cycle (PDCA’s Beginning, 2006; Lovitt, 1997).

The PDCA Cycle encourages teachers to be methodical in their lesson planning, and to discuss and refine their teaching for subsequent improvement of future lessons. This method requires the teachers to (1) PLAN a lesson, (2) DO or teach the lesson, (3) CHECK or discuss ways to improve the lesson, and (4) ACT or repeat the lesson with the suggested changes. In the PDCA Cycle, a group of teachers meet as a team to set goals and to collaboratively design a lesson. Once the lesson is crafted one or more of the teachers in the group teach the lesson, while the other group members observe the lesson, occasionally acting as students. After the lesson the whole group meets to analyze and reflect on what transpired as well as to discuss ways to re-teach the lesson as a means of making it more effective and motivating for the students. Then the revised lesson is executed, observed, and reflected upon as a means of completing the cycle.

Sallis (2005) along with Chokshi and Fernandez (2005) believe that the PDCA Cycle provides an important springboard for the professional development of teachers. Not only does it develop professional knowledge among both inservice and pre-service teachers, it also demonstrates the connection between educational policy and practice. Hiebert and Stigler (2000) concur with this notion and describe the cycle as a teaching improvement system based on research and the notion that teaching is a multifaceted, cultural activity. The present study endeavored to provide pre-service teachers an opportunity to gain a practical understanding of this important springboard for their own professional development.

Preparing for a PDCA Cycle
As part of their senior-level mathematics education seminar, twelve pre-service mathematics teachers were given the opportunity to design and implement an actual PDCA Cycle study. Prior to preparing an instructional unit on mathematical relations, the pre-service teachers reviewed research literature on teacher improvement processes and wrote essay papers that outlined their goals, objectives, and expectations for the study. In addition, the future teachers were encouraged to express their personal concerns before undertaking the PDCA Cycle.

The teacher candidates highlighted four areas within their essay papers, (1) their desire to collaborate effectively within each study group, (2) their ability to motivate
students to partake in the instructional activities, (3) their ability to enable students to clearly comprehend the instructional content, and (4) to provide insights on how the pre-service teachers could learn how to evaluate their own and other’s teaching. Within the essay papers the teacher candidates wrote about their expectations of viewing different types of effective teaching methods and of their anticipation of using the PDCA process to learn how to adapt and revise a lesson before re-teaching it. The pre-service teachers also expressed reservations about the time-consuming nature of the PDCA process.

In addition, the future teachers who were scheduled to teach the first lesson related their anxiety in teaching a topic that they did not feel they fully grasped, while the future teachers who were scheduled to teach the second (a revision of the first) lesson wrote of their desire to improve the quality and presentation of the lesson and to use the PDCA experience to make themselves better prepared teachers.

Participants
Three classrooms of pre-service teachers participated in the study, and in this report, pre-service teachers, prospective teachers, and future teachers, all refer to the same cohorts. One classroom of 12 fourth-year prospective secondary mathematics teachers designed, implemented, and served as the instructors for the PDCA study. Two other classrooms (each of 30) second-year future primary school teachers who were enrolled in the first mathematics course (from a two-course sequence) participated as the students in the investigation. The consideration and selection of pre-service teachers as ‘instructors’ and ‘students’ for this PDCA study gave the prospective teachers an opportunity to participate in the type of study that was apt to be part of their professional futures.

Process
During their mathematics education seminar, four of the future teachers jointly prepared a lesson plan on relations and equivalence relations that included the lesson’s content within the Common Core Standards for Mathematical Practices (Common Core State Standards Initiative, 2014), teaching procedures, worksheets, and an assessment plan. Two of the pre-service teachers then taught the lesson to the first classroom (Group 1, n=30) of prospective primary school teachers. The topics of relations and equivalence relations were selected because they are an integral part of the mathematical preparation for prospective secondary school mathematics teachers, as well as a common part of the content in textbooks for prospective primary school mathematics teachers.

The lesson was videotaped and later viewed by all twelve of the pre-service mathematics teachers who were enrolled in the mathematics education seminar. Following the viewing, these prospective secondary mathematics teachers wrote reaction papers to the lesson. During a subsequent class the future teachers discussed ways to improve the lesson based on results from the in-class worksheets, assessments, and from debriefings by the two participants who taught the lesson.

Four other seminar participants neither of whom had written the first lesson plan nor taught the first lesson, prepared a second, revised lesson plan and two of them taught the lesson to the second classroom (Group 2, n=30) of prospective primary school teachers. After the second class was filmed, all twelve prospective secondary mathematics teachers viewed the film and wrote reaction papers. The film of the class instruction and
the pre-service teachers’ reactions were discussed during the next seminar class meeting. The class ended with the teacher candidates being given the assignment to write a summary paper of their overall views on the PDCA study and on their individual experiences during the PDCA Cycle.

First lesson
The instructional materials used during the first lesson included an interactive whiteboard presentation and described a relation on a set X as “any set of ordered pairs in which the first and second components are from X.” Reflexive, symmetric, and transitive properties of a relation were also described for a relation on a set X, and an equivalence relation was described as any relation on X that satisfies these three properties. The students were also given a worksheet that included three tasks to complete.

During the first lesson students were given the definition of a relation and an explanation of its properties. The students were then given an example of a relation and asked to indicate whether the given relation on the set of all people was reflexive, symmetric, or transitive. They were then instructed to summarize their answers for the following phrases (see Table 1).

Table 1. Summary table for relation problems

<table>
<thead>
<tr>
<th>Relation</th>
<th>Reflexive</th>
<th>Symmetric</th>
<th>Transitive</th>
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</thead>
<tbody>
<tr>
<td>“is a descendant of”</td>
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<tr>
<td>“is a different weight than”</td>
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<tr>
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The students were given a second activity sheet, which asked them to determine if the relations given by three arrow diagrams were reflexive, symmetric, or transitive (see Figure 1.)

![Figure 1: Arrow diagram activity](image)

For the third activity sheet, the students were instructed to draw arrow diagrams within ovals that showed relations that were: (1) reflexive and symmetric, but not
transitive; (2) reflexive and transitive, but not symmetric, and; (3) symmetric and transitive, but not reflexive.

Second lesson
Definitions of the reflexive, symmetric, and transitive properties as well as a definition of a relation and of an equivalence relation were included in the revised second lesson plan. After considering comments made by the prospective teachers during the follow-up discussion in their seminar class, a few changes were instituted. The second lesson plan included an interactive whiteboard display containing a definition of a relation along with four examples of the relations: “has the same decimal value as,” “is less than,” “x^2 + y^2 = 1,” and “x + y < 1.” For the relation “has the same decimal value as,” an ordered pair member of the relation was displayed on the interactive whiteboard with the statement “(1/2, 3/6) is in the relation.” For the relation “is less than,” the example was the statement “3 is related to 5 but 5 is not related to 3.” For the relation “x^2 + y^2 = 1,” the example included the statement that “(1, 0) is in the relation, but (2, 2) is not.” For the relation “x + y < 1,” the example contained the statement “(-1, 0) is in the relation, but (2, 5) is not.” A table similar to Table 1 was used except that the relation “is older than” replaced “is a descendant of” (see Table 2). Also included with the second lesson plan were the same worksheets that dealt with reading and constructing arrow diagrams.

Table 2: Revised Summary Table for Relation Problems

<table>
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Results
At the conclusion of both lessons on relations, Group 1 and Group 2 of the pre-service primary school teachers completed test items that used arrow diagrams to represent the reflexive, symmetric, and transitive properties. The test items specifically asked these future teachers to illustrate situations that were 1) symmetric and transitive, but not reflexive; 2) reflexive and symmetric, but not transitive; and 3) reflexive and transitive, but not symmetric. The results showed that the pre-service teachers successfully responded to the test items with illustrations that had arrow diagrams.

The results from Group 1 demonstrated that 65% of the students illustrated a situation that was symmetric and transitive, but not reflexive. In addition, 45% of Group 1 students illustrated a situation that was reflexive and symmetric, but not transitive; while 8% illustrated a situation that was reflexive and transitive, but not symmetric.

The results from Group 2 demonstrated that 35% of the students illustrated a situation that was symmetric and transitive, but not reflexive. Half of the Group 2 students correctly illustrated a mathematical situation that satisfied the reflexive and symmetric properties, but not the transitive property. Forty percent of Group 2 students illustrated a situation that was reflexive and transitive, but not symmetric.
Students’ comments of the lessons
Students from both Groups 1 and 2 were asked to give an overall assessment of the lesson and to specifically write comments pertaining to the lesson on relations, their properties, and equivalence relations. In addition, they were asked to comment on what aspects of the lesson had been most helpful. When considering which aspect was most helpful, 50% of the participants from Group 1 responded favorably to the diagrams; while 20% noted the examples and 20% noted the explanations on the interactive whiteboard presentation. The most helpful aspects of the lesson from Group 2 were the diagrams (40%), the examples (23%), and the interactive whiteboard explanations (18%). It should be noted that 75% of the students from both Group 1 and Group 2 rated the class instructional activities as being very good.

Prospective teachers’ comments of their experiences with the PDCA Cycle study
After viewing the second lesson, the pre-service mathematics teachers were asked to write an in-depth reflection paper describing their experiences with the PDCA Cycle and to include any overall comments they had about the study. The results indicated that the prospective teachers valued: (1) the opportunity to experience the PDCA study process within an academic classroom, (2) the opportunity to work with their peers to develop and improve a potential lesson, and (3) the experience of observing, and in a few cases teaching a lesson that had been developed as a group initiative. The future teachers also commented that it was beneficial to observe others teach as well as to view and discuss the videos of the lessons.

However, several of the pre-service teachers expressed some dissatisfaction that the lessons did not result in more student participation and interactions, while other pre-service teachers commented on the preparation and delivery of the examples in the first lesson. These same pre-service teachers pointed to the fact that during the initial preparation of the first lesson, a few of them addressed the examples as being too confusing for the students and called for clearer illustrations. Other suggestions for the improvement of the lessons were for better-prepared interactive whiteboard displays, handouts with definitions, for more clarity in the initial examples, and for clearer explanations of relations and their properties.

In general, the majority of the prospective teachers indicated that they would include more and differing types of examples in the lesson. That is, they would include more mathematical examples as well as clearer examples of relations and equivalence relations. Other comments included increasing class participation by asking the students to construct their own examples.

Conclusion
The pre-service teachers involved with this project developed strong working relationships within their groups that allowed them the opportunity to collaborate and reflect on their mathematics lessons and teaching. The prospective teachers not only exhibited collegial qualities that supported learning, but they enhanced their teaching improvement skills by following a cohesive PDCA Cycle that included both mathematical and pedagogical knowledge.
In addition, participating in the PDCA project helped the teacher candidates to systematically develop their understanding of recent educational reform. Discussion of educational reform constituted a sizable component of the seminar’s content, but the experience of implementing classroom reform was very beneficial for the pre-service teachers as they concretely encountered reform issues through their involvement with the PDCA activities. This was evident in their written commentaries of the analysis of the videos and lesson materials and when the future teachers witnessed that the participants who taught the lessons were more prone to tell their students about relations without providing tasks for the students to conjecture and construct their own relations. As the pre-service teachers further noted in their commentaries, it is vital for students to be able to construct their own knowledge through mathematical reasoning and experimentation.

The prospective teachers’ involvement in the PDCA study was not without challenges. Most of the teacher candidates felt anxiety with the process of joint lesson planning, but embraced the process of learning how to apply skills such as posing researchable questions, designing classroom experiments, specifying types of evidence for collection, and interpreting and generalizing results. In particular, some of the teacher candidates expressed an anxiety of being filmed and making their teaching public.

If we are to follow the philosophy of Moen and Norman (2010) and other advocates of this methodology (Tague, 2005; Yoshio, 1993), then the PDCA Cycle will remain relevant and will continue to evolve in the future. This project demonstrated that a PDCA study is not just about collaboratively planning, teaching, and improving a single lesson. It’s about developing a pathway for continual enhancement and creative endeavors in one’s teaching repertoire.

References

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