A study of student-centred activities for low attainers in primary mathematics
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Introduction
Kaur and Ghani (2012) noted that low attainers in mathematics preferred to work in
groups on mathematical tasks with the help of manipulatives. They found interacting
with peers a fun and good way to learn, and the “hands on” experience gratifying and
meaningful in learning. However, observation of lessons of these pupils showed that
there is a mismatch between how teachers teach these pupils and how these pupils
would like to be taught in mathematics lessons. An intervention project, pedagogy for
low progress learners of mathematics, is presently underway in some schools in
Singapore. In this paper we study the lessons of two teachers in the project with a focus
on the activities in their classrooms. We examine the activities for their purpose and
intended learning.

Literature review
In the context of viewing learning as a generative process of meaning-making and
mathematics as a dynamic discipline, student and teacher engagements in mathematical
activities are possible sites for student struggles (Warshauer, 2011). Warshauer
examined activities and interactions that facilitate struggle as a productive part of
mathematics learning and understanding. Warshauer refers to students’ productive
struggle as their “effort to make sense of mathematics, to figure something out that is
not immediately apparent” (Hiebert & Grouws, 2007, p. 287) as opposed to students’
effort made without direction or purpose. Productive struggle is a particular kind of
phenomenon that directs the process of students’ struggle towards understanding,
reasoning, or sense-making of the mathematics with possible support from the teacher
or peers and gives students a sense of agency in doing mathematics (Kilpatrick,
Swafford, & Findell, 2001). In other words, there are signs of productive struggle when
students who were struggling indicate a better sense of what to do to get started with a
problem, how to carry out processes, or why a problem and its solution make sense. In
other situations, students are better able to reconcile a misconception, explain or justify
their work, determine an error in their work, or recall factual information useful for their
task (Warshauer, 2011). This is in contrast to what Warshauer identified as
unproductive struggle, a phenomenon in which students who show signs of struggle
make no progress towards sense making, explaining, or proceeding with a problem or
task at hand. A student may voice resignation and give up, take up another task, or
obtain an answer from a teacher or student, thereby removing the struggle but not
productively building mathematical understanding.

Findings from Warshauer’s study provide evidence that there are aspects of
student-teacher interactions that appear to be productive for student learning of
mathematics. The encouragement to communicate with teacher responses such as, “Tell
me what you mean” and “Talk about it some more” or insistence on sense-making with “Why is that?” provided opportunities for students to elaborate on what they understood and perhaps clarified the source of their struggles. Responses that encouraged continued effort such as, “Try that” and “Well, what if you do...” gave positive reinforcement for engagement without student worrying about whether the result was right or wrong. An appropriate tempo for the interaction that did not rush the process or resort to shortcuts promoted the sense that understanding both the problem and the process was more important than just finding a quick way to finding the answer.

In this paper, we attempt to extend the findings of the study on Low Attainers in Primary Mathematics (LAPM), which was carried out in Singapore (Kaur & Ghani, 2012). It is apparent from the findings that the challenge for teachers of low attainers in mathematics is to provide instruction comprising a good balance of “how they would like to learn” type of activities whilst engaging them in productive struggle.

The study
The study reported in this paper examined the lessons of two teachers participating in an intervention project, pedagogy for low progress learners of mathematics, with a particular focus on the activities, which the teachers enacted in their lessons. The three research questions that guided the study were as follows:

1. What are the predominant types of activities (teacher-centred versus student-centred) present in the lessons of the two teachers?
2. What is the nature of student-centred activities in the lessons of the two teachers?
3. Do the student-centred activities engage students in productive struggle?

Participants
The participants of the study were two teachers, Teacher 1 (T1) and Teacher 2 (T2) who participated in the project. T1 and T2 taught the topics of ‘Percentage’ and ‘Time’ respectively, across a sequence of four lessons each to grade 5 Foundation course students. There were about twenty pupils, both boys and girls, in each class.

Data and analysis
The data comprise video-records of sequences of lessons taught by T1 and T2 for the topics ‘Percentage” and ‘Time’ respectively. The first task undertaken was to develop a comprehensive set of codes for coding the video data. The coding was carried out as follows.

Part 1
For both the teachers, the first viewing of the video-recorded lessons was to mark out all the different teaching and learning activities that took place during the lessons. The video records were segmented into one-minute episodes/intervals. Next, each episode was colour coded based on the identified instructional practices that were prevalent in the interval. While coding, specific examples of the teaching/learning activities present and the main concepts taught were identified. The set of questions for the data analysis facilitated the descriptive analysis of the activities. The authors carried out the viewings independently, following which, the marked out segments were compared, and through discussion, the activities were classified. In the context of this paper, an activity is defined as a teaching/learning episode during which, pupils engage in any of the following: individual seatwork, group work, pair work or whole class exposition. The
activities may involve student practice, students working with manipulatives and/or paper and pencil tasks in groups or individually; teacher explaining concepts, demonstrating procedures or evaluating student understanding via IRE (Initiate, Response, and Evaluate) iterations (Mehan, 1979). For the purpose of our investigation we first categorized the activities into two main types. They are:

1. Student-centred activities: These comprise activities, other than seatwork (where students work independently on teacher produced worksheets or exercises from workbook), such as students working in groups, pairs or individually to make sense of the mathematical tasks that they were given by the teacher. Students may be provided with manipulatives to work with and complete a worksheet or merely be given a worksheet to complete, which facilitate explicit articulation of thinking processes by students, while the teacher circulates among the groups to ask targeted questions or to facilitate discussion. They may also be asked to present their reasoning and understanding arising from the task to the class.

2. Teacher-centred activities: These comprise activities during which, students passively receive information from the teacher who actually does most of the showing and talking, for example, students learn through listening and observation during teacher exposition and recapitulation of the concepts learnt.

The third types of activities, which could not be categorized as either type of the above two, are Interruptions/Downtime/Non-curriculum oriented activities such as, lesson organization/distribution of resources and when the teacher is disciplining a student.

Part 2
In the second viewing of the video-recorded lessons of the two teachers, we focused only on the student-centred activities other than seatwork that took place during the lessons. To classify the nature of student-centred activities, we created a framework. The framework comprising three levels is shown in Table 1.

<table>
<thead>
<tr>
<th>Level</th>
<th>Focus of student-centred activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>To capture the attention of students and motivate them</td>
</tr>
<tr>
<td></td>
<td>To engage students in making sense of mathematical concepts</td>
</tr>
<tr>
<td></td>
<td>To support students in their productive struggle</td>
</tr>
<tr>
<td>2</td>
<td>To capture the attention of students and motivate them</td>
</tr>
<tr>
<td></td>
<td>To engage students in developing procedural fluency of mathematical skills</td>
</tr>
<tr>
<td>1</td>
<td>To capture the attention of students and motivate them</td>
</tr>
</tbody>
</table>

Examples of activities at the three levels are given below
Example 1 shows such an activity that was categorised as Level 3.

Example 1: T1 Lesson 2 Activity 2.2 – “The Number game” (Pair Work)
During this activity, the teacher advanced students’ understanding of number bonds. He engaged them in making 100 using multiplication/division. At the beginning of the activity the teacher showed examples of number bonds (as shown in Figure 1) on the whiteboard. Next, students were given the number cards (manipulatives) for seatwork
(Figure 2). The teacher gave explicit instructions for the activity. Students worked in pairs to make the number bonds using the cards.

It was evident that students experienced ‘productive struggle’ during the activity. The teacher provided support whilst students were struggling. For several pairs of students the teacher was seen providing support by engaging them in IRE iterations till they were able to manage the task. At times the teacher was also heard asking some students to demonstrate their solutions on the whiteboard after they have solved them. He also encouraged the students to compare alternative methods, for example they could use addition to prove that 100 times 2 gives 200 (“I need (to add) 2 100s to get one 200”) and 20 x 5 = 100 (“adding 20 five times, gives 100”). The students were also asked to articulate their reasoning as the teacher checked for possible misconceptions. The teacher was heard asking his students “Why is it that…?” when they showed him their number bond models.

![Figure 1. Teacher models problem 1 using a diagram](image1)

![Figure 2. The number game](image2)

Example 2, shows an activity categorised as Level 2.

**Example 2: T2 Lesson 3 Activity 3.3 – “Time game” (Pair Work)**

This activity focuses primarily on the students’ procedural fluency/efficiency. The purpose of this activity was to practice and review the concept of telling/showing time through a game (See Figure 3). At the beginning of the activity, the teacher instructs the pairs of students to number themselves 1 and 2. Next the students take turns to do the following: one of them shows the time while the other reads it aloud. A correct response gets 1 score point. The teacher checks scores of each pair after the activity and praises them. As the students play this game in pairs, this activity allows them to interact with others and learn from their friends.

Although the teacher was successful in providing a motivating environment throughout the activity and the students achieved the learning objective of telling/showing time, the activity did not pose any productive struggle for the students, as they were able to tell/show time with minimum difficulty. It could be because the students have already mastered the concept of telling/showing time in the first two lessons.
Example 3 shows an activity categorised as Level 1.

**Example 3: T1 Lesson 1 Activity 1.2 - “Think, Puzzle, Explore” (Group activity)**

The ‘Think, Puzzle, Explore’ activity required the students to write any question that came to their mind about percentage. The teacher used manipulatives (charts, colour markers) to assist with the expression of ideas. Before introducing the activity, the teacher spent 15 minutes talking about the topic (percentage). He gave various real life examples, such as, advertisements in newspapers, flyers, shopping malls, a cartoon clip showing a boy who did not give his 100% effort in the game, marks in exams, percentage of gases in the atmosphere, etc. He tried to build confidence in students so that they did not hesitate in asking questions. The teacher often guided the students during the activity to think and come up with possible examples in real life about the use of percentage. For example, he said, “think about the drop in the number of car sales. Did they use percentage?” He also used teacher talk to focus/re-focus students’ attention as he reminded them during the activity, “write as many words as possible,” “what do you think after looking at the picture?” The teacher guided the students in thinking of possible questions for the ‘Explore’ activity by prompting, “for example, ‘who came up with percentage’ is a very good question.” He encouraged them to think out of the box and to come up with different questions about percentage. At the end of the activity, students got an opportunity to view the work of other groups through a ‘gallery walk.’ The teacher encouraged students to put a ‘star’ or a comment on the questions that they liked or did not. Although this activity is motivating and involves collaborative work it does not explicitly advance any conceptual or procedural forms of mathematical knowledge.

**Results and discussion**

In this section, we present the findings of the study and answer the three research questions that guided the study.
Question 1: What are the predominant types of activities (teacher-centred versus student-centred) present in the lessons of the two teachers?

Table 2 shows the frequency of the occurrences of the two types of activities for sequences of lessons of T1 and T2. From the table it is apparent that T1 incorporated a higher percentage of student-centred activities (81.8%) in his lessons when compared with T2 (41.7%). It may be said that in the class of T1 the predominant type of activity was student-centred while that in the class of T2 was teacher-centred.

Table 2. Frequency of activities

<table>
<thead>
<tr>
<th>Type of Activity</th>
<th>Frequency N (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>T2</td>
</tr>
<tr>
<td>Student-centred</td>
<td>9 (81.8%)</td>
</tr>
<tr>
<td>Teacher-centred</td>
<td>2 (18.2%)</td>
</tr>
<tr>
<td>Total</td>
<td>11 (100%)</td>
</tr>
</tbody>
</table>

Question 2: What is the nature of student-centred activities in the lessons of the two teachers?

Table 3 shows the frequency of student-centred activities at the three levels. It is evident from Table 3 that T1 enacted activities at all three levels with almost half of them merely for the purpose of engaging and motivating the students. In contrast, T2 only enacted activities at levels 2 and 3.

Table 3. Frequency of student-centred activities at levels 1, 2 and 3

<table>
<thead>
<tr>
<th>Level of student-centred activity</th>
<th>Frequency N (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T1</td>
</tr>
<tr>
<td>1</td>
<td>4 (44.4%)</td>
</tr>
<tr>
<td>2</td>
<td>2 (22.2%)</td>
</tr>
<tr>
<td>3</td>
<td>3 (33.3%)</td>
</tr>
<tr>
<td>Total</td>
<td>9</td>
</tr>
</tbody>
</table>

Question 3: Do the student-centred activities engage the students in productive struggle?

From the qualitative analyses of the data (teacher-student responses), it is reasonable to suggest that the student-centred level 3 activities enacted by T1 and T2 do engage students in productive struggle. For example, there is evidence of students experiencing ‘productive struggle’ during an activity when they try to sort out the correct number cards to make number bonds using multiplication/division. The teacher facilitates supportive intervention during the activity and insisted on sense-making by providing prompts such as “If I show you 2, what will you show me?” and “How do you know that?” instead of directly providing the answers to the students. This helps in optimizing the learning opportunities embedded in students’ work and productively building their mathematical understanding. This finding concurs with what Warshauer (2011) stated in his study that communicating with teacher responses such as, “Tell me what you mean”
or insistence on sense-making, provide opportunities for students to elaborate on what they understood and perhaps clarify the source of their struggles, thereby giving them a sense of agency in doing mathematics (Kilpatrick, Swafford, & Findell, 2001). It also promoted the sense that understanding both the problem and the process was more important than just finding a quick way to finding the answer.

The findings also showed that most of the level 1 and level 2 activities are engaging and motivating for the students but do not impart substantial mathematical knowledge, nor do they allow any productive struggle in further developing the mathematical concepts. The classroom environment was warm and the teachers attended to all the difficulties of their students during between-desk instructions. However, the teachers oversimplified and scaffolded more than necessary during these learning episodes, thereby depriving the students of the intellectual effort required by the activities. More attempts could have been made by the teacher to engage students in articulating their thoughts and reasoning about the mathematics they were working with.

**Conclusion**

The study shows that both teachers did incorporate a variety of student-centred activities in their lessons for encouraging student participation and motivation. However, it was observed that there is a lack of opportunities for productive struggle during these activities. Teachers need to plan more student-centred level 3 activities in their lessons, which are not only engaging and motivating but also involve students to make sense of the mathematical knowledge that is embedded in the activities. Therefore in attempting to strike a balance between how students would like to learn and how teachers may engage them in meaningful learning of mathematics, we propose that teachers of low attainers incorporate activities of level 3 in their lessons to teach or revisit concepts.

We also suggest that teachers engage students in productive struggle and refrain from oversimplifying tasks as Watson (2001) found that low attaining students are capable of making shifts in their thinking from superficial features of a mathematical task to forms of mathematical thought. Watson asserts that low attainment is not the result of an inability to think but the lack of structured work that promote higher order thinking.

**References**


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