

How the first grade pupils express models: Focus on reformulation of situation models in a modeling teaching

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Introduction

In this paper I introduce modeling teaching practices for the first grade pupils in a public primary school in Japan and report facts about them, and discuss a reformulation of situation models. I focus on the dissimilarity between models, and analyze situation models that pupils create with referred utterances by the teacher and the pupils.

Reformulation of situation models

Blum and Leiß (2007) indicated that there are reciprocal processes between a real situation and a situation model in a modeling cycle (*Figure 1*), and explained that this process is the most important process as a phase of understanding a modeling task.

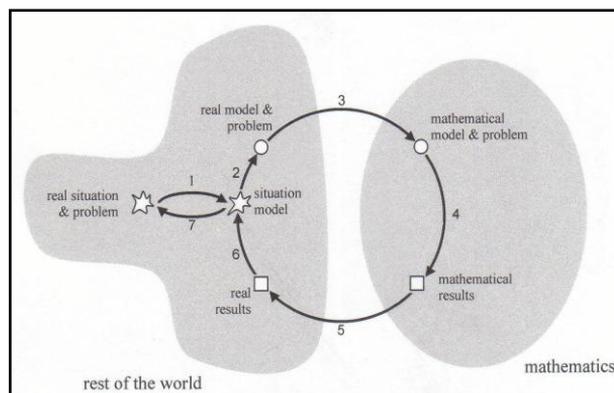


Figure 1. Modeling cycle (based on Blum & Leiß, 2007, p. 225)

Borromeo Ferri (2006) focused on differences of individual mathematical thinking style, and explained the presence of these processes through distinctions between the situation model/mental representations of the situation and the real model. Some previous studies of modeling propose the method to visualize modeling progress. Borromeo Ferri (2007) focused on the mathematical modeling progress of individual modelers calling their overall attempts 'modeling routes.' Grigoraş et al. (2011) focused on the process of mathematizing and indicated paths of epistemic action based on two different frameworks of structural actions and Anthropological Theory of Didactics. Finally, Matsuzaki (2011) identified components of the modeling based on each modeler's prior experiences and displayed modeling progress using applied task analysis mapping. From the perspective of metacognition, Stillman (2004) discussed the reformulation of models related to real situations. Matsuzaki and Kawakami (2010) discussed the reformulation of situation models through setting up problems from the task situations in the real world.

Purpose and method

In this paper, I develop a modeling task and implement a modeling teaching practice, and discuss evidences of reformulations of situation models.

Developing a modeling task

In previous studies, modeling problems related to real situations were reported (Matsuzaki, 2011; Matsuzaki & Kawakami, 2010). Findings indicate that modelers require a variety of experiences for solving, and we can confirm the experiences from pictures or protocols as one of the evidence.

From an old mathematics textbook in Japan

There was a section ‘placement of the things and its expressions by figures’ of the unit ‘ways to express in a figure’ in a mathematics textbook for middle and high school women in Japan. In the section, a problem of arranging tableware is as follows:

A figure shows an example of arranging tableware. Devise various ways to arrange the same group, for example, coffee cups or slipper and so on, and express them by simple figures.

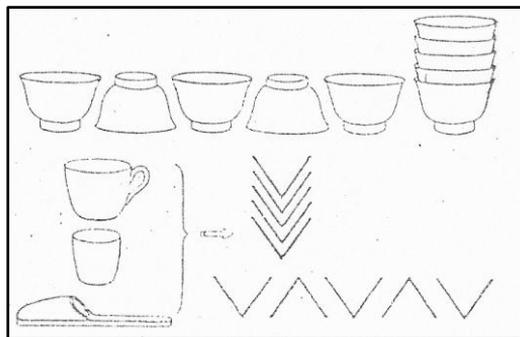


Figure 2. Japanese old textbook
(Cyuutohgakkoukyokasyo-Kabushikigaisya, 1943, p.33)

Research for undergraduate students and its results

I implemented a research with undergraduate students who would like to be mathematics teachers after graduation. I presented pictures of two types of school lunch (Figure 3), and asked them the problem: ‘How do you tidy up the tableware for four friends by yourself? Draw pictures or figures.’

In type (A), the tableware includes two kinds of devices (one dish plate and soup bowl) to serve on and two kinds of cutleries (spoon and fork). In addition, there are one milk pack and packaged bread rolls. In type (B), the tableware includes three kinds of devices (soup cup, dish plate, and desert plate) to and four kinds of cutleries (knife, fork, spoon, and small spoon). The responses below are from two students, IJ and HT.



Figure 3. Two types of school lunch

Student IJ drew pictures by considering the shape or size of the devices, and put tools in a bowl. He especially pointed out that milk packs were squashed flat and the packages were tied in type (A) lunch. Although he did not pile up soup cups, which were one of the devices in type (B), he pointed out that the shape of each soup cup included the handles, and put them on plates piled up or were used to put the cutleries.

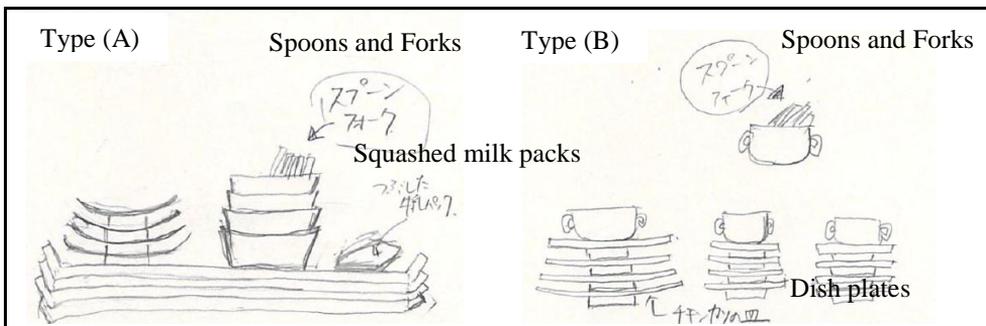


Figure 4. Responses by undergraduate student IJ

The student HT drew pictures through consideration of same shape/depth of devices, and show leftovers on the top of devices. Although his expression of tools were different between two types. In the type (A) he used a same symbol and explained each tool to distinguish supplementary. In the type (B) he pointed out that there were not tray and gathered tools were put on the biggest plate.

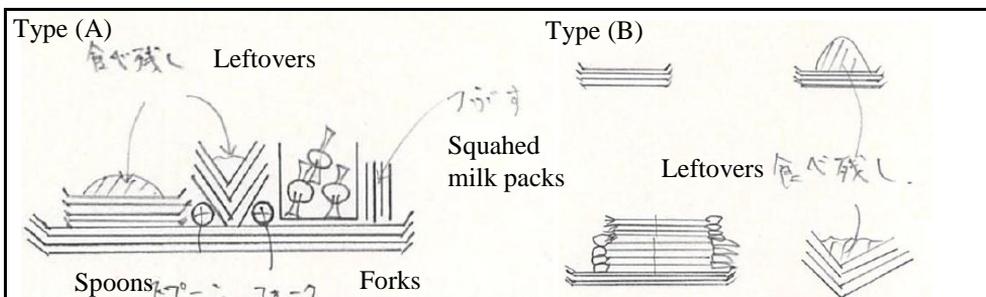


Figure 5. Responses by undergraduate student HT

In this way I find that the students above used variety models and expressed them for tidying up tableware. As shown these pictures, the students may simulate of tidying up tableware. It is said that these models may be one of evidences of reformulation of situation models. So I develop a modelling problem referring the problem above as follows: ‘*How do you tidy up school lunch of four friends by yourself?*’ located on ‘real situations and problem’ in modelling cycle (see Figure 1). Here I set school lunch situation because it is easy for the pupils to share the situations. It may be not easy for infant pupils to express model(s) on worksheets. In addition, I adopt manipulatives which are paper devices and plastic spoons (see also left of *Figure 6* in this page) as the modelling task. I hope that they will be able to express model(s).

Flow of Lesson

A modelling lesson was implemented for a 1st grade class of twenty-four pupils at a public primary school in Saitama prefecture. This lesson was implemented for 45 minutes on 7th July 2014. The teacher is Matsuzaki, and the lesson was recorded by digital video cameras and voice recorders.

Situation of the School Lunch

The teacher presented a picture of the school lunch to the pupils, and he and his staff pretended as if they serve meals of the school lunch. He indicated the pupils to display paper devices and plastic spoon in the same position as the picture, and asked them difference between the picture and tableware.



Figure 6. A school lunch and set devices (paper bowls/plate, handicraft board, and plastic spoon)

The pupils said as follows: No milk pack. There are two partitions on paper plate, but one partition on real plate. No painting on paper bowls/plate. Sizes of the real and paper devices are different. Two paper bowls are same shape and size, and there is an affix yellow label on a bowl to distinguish the bowl for rice and carry soup. Here the teacher presented the modelling problem.

Explanations of Tidying Up Tableware

In this class there are no experiences to tidy up tableware by gathering devices and tools. The pupils were indicated to write down their own ideas on worksheet, and they presented ways to tidy up tableware using manipulatives (paper bowls/plate, craft boards, and plastic spoons) within each group. Next, they explain their own way to mates of the group using manipulatives as a modelling task. After that, they discussed the best way of tidying up tableware. The teacher called on three groups to explain the ways through the discussion.

The First Group

Hono explained the way of tidying up each device and tool to mates within the group using paper devices and plastic spoons as follows;

30:11 *Hono*: I put the first person's tableware in order..., and the second person's one..., and the third person's one..., and the fourth person's one.



Figure 7. Explanations by Hono

In explaining the way of the first group, Hono made a gesture of tidying up tableware.



Figure 8. Explanations the way of the first group to all the pupils

38:00 *Teacher*: Please explain your ways to all the pupils.

38:28 *Hono*: I put the first person's tableware in order..., and next person's one..., and next..., and next.

38:48 *Hiro*: Your explanations are different from explanation for our group!

39:00 *All*: Not understandable!

The Second Group

The following protocol was exchanged between the teacher and the pupils when the teacher was walking around this group;

34:35 *Ari*: Everyone's ways are same...four all.

34:37 *Teacher*: Are everyone's ones same?

34:38 *Ari*: [Nod] Yes.

The pupils of this group tidied up every kind: paper bowls for rice, paper bowls for carry soup, paper plate, and plastic spoons. The pupils categorized four groups that were three devices and spoons.

42:11 *Ari*: I want one more bowl for carry soup.

42:21 *Ryo*: I want one more spoon, Ari.

42:41 *Ari*: All finish?

42:43 *Ryo*: Yes....(counting spoons) 1, 2, 3, 4.



Figure 9. Discussion within the second group

In explaining the ways by the second group Masa who has paper bowls for carry soup said by presenting devices of paper bowls, paper plates and plastic spoons.



Figure 10. Explaining the ideas of the second group to all the pupils

41:30 *Masa*: We put each tableware in order.

41:38 *Teacher*: What is different from Hono's explanation?

41:43 *Hono*: They put each kind of devices and tools in order.

The Third Group

The pupils of this group also tidied up every kind: paper bowls, paper plates, and plastic spoons. The pupils categorized three groups that were two devices and spoons. And they piled up craft boards, and put devices and spoons on the boards through discussion within the group.



Figure 11. A scene of piling up craft boards through group discussion
When the pupils explained to all the pupils, they did not bring craft boards.



Figure 12. Explaining the ways of the third group to all the pupils

The teacher asked for all the pupils, and the pupil Ato responded as follows;

44:28 *Teacher*: Is this idea a same idea of the way of the second group?

45:02 *Ato*: In this idea bowls for carry soup and bowls for rice are put all together.

Analysis of Models on Worksheets

The pupils were instructed to write down their own idea on worksheet in individual work. They expressed by description(s) and/or picture(s) to the problem. Only one pupil was halfway through description, and no response was two pupils.

Models expressed by description

Responses by only descriptions are total twenty-one pupils.

Firstly, for example, a response as 'I tidy each one up' is nine pupils, but the response is not clear the way of tidying each one up shown in the first group. There is an additional response as deciding allotment. Secondly, for example, a response as 'I pile up tableware' is seven pupils, but the response is not clear the way of piling up devices and gathering spoons shown in the second group and the third one. Thirdly, another response is to tidy up tableware not by yourself.

As a result, it is too difficult to specify situation model(s) and chase reformulation of them from responses of only descriptions.

Models expressed by pictures

On the other hand, three pupils drew pictures only.

Firstly, a picture drawn by Rika is the situation of tidying tableware up.

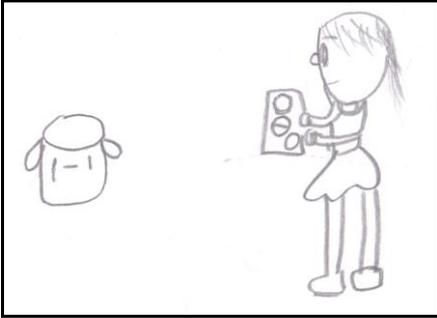


Figure 13. A Picture drawn by Rika

This picture seems like a usual scene of tidying up tableware of school lunch (*Figure 14*), and here this situation model is similar to usual situation model. Because the way of tidying up each device and spoon, and it may be the way of tidying each one up. Although it is not able to say certainly that the way of Rika's is not the way of piling up devices and gathering spoons, reformulation of situation models is not able to be specified only from this picture.



Figure 14. A usual scene of tidying up tableware of school lunch

Secondly, a picture drawn by Rin shows the way of piling up devices and gathering spoons. The features of this picture are viewed from right above, and the situation of piling up tableware with three/four-layered lines or curves. Although we are not able to see the situation of piling up tableware viewed from right above, each device or spoons were simplified/structured that was a process from situation model to real model, and this picture located on real model. Thus constructed situation model is different from usual situation model, and it is said that reformulation of situation models is occurred.

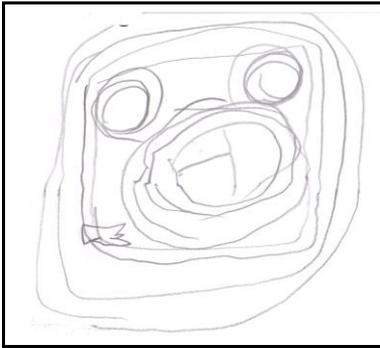
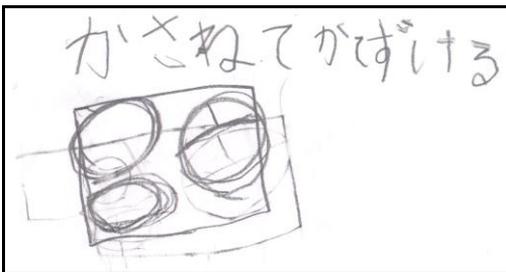


Figure 15. A Picture drawn by Rin

Thirdly, only one pupil, Rik, drew picture with the description (Figure 16). The description represents the way of piling up devices and gathering spoons, it is located on real situation. This picture also shows the way of piling up devices and gathering spoons. The features of this picture are viewed from above, and layered curves seems bold curves, too. Each device was simplified/structured, and this picture located on real model. Thus constructed situation model is different from usual situation model and explain real situation supplementary. Namely, it is said that reformulation of situation models is occurred.



I pile up
tableware.

Figure 16. A Picture with description by Rik

Conclusions

In this paper I developed the modelling problem with referring old mathematics textbook of middle and high school for women in Japan. And I implemented the modelling teaching practice for the first grade pupils adopted with manipulatives as the modelling task. Most of the pupils were embarrassed and it is difficult for them to write down ways of tidying up devices and spoons on worksheet. It is not difficult for them to exchange ways using manipulatives in each group. Although I can not analyze reformulation of situation models from protocols or manipulation as the modelling task. In addition, models expressed by description are not enough to specify situation models. On the other hand, models expressed by pictures show reformulation of situation models. For the first grade pupils in Japan mathematics teaching to promote concepts formation of the figure are the first step. Especially, teaching about plane figures and space figures are not conducted on the first semester. The figures seen in research for the undergraduate student represent mathematical models (see Figure 5), it is possible to express models mathematically. Thus it is possible to simulate to tidy up tableware in this modelling problem, we hope to analyze reformulation of situation models as one of evidences. I would like to continue research and implement lessons using this modelling

problem. At that time, it is needed that considering learning contents and developmental levels of pupils/students that most of them can express models mathematically.

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