Developing PISA-like mathematics tasks for investigating Indonesian students' profile of mathematical literacy

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Introduction

The issue of mathematical literacy has been discussed, mainly since the PISA survey was firstly conducted in year 2000. A feature that does emerge to differentiate mathematical literacy from mathematics is an emphasis on the use of mathematics in context (OECD, 2013; Steen, 2001). PISA mathematics framework 2012 defines mathematical literacy as a person's ability to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognize the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens. (OECD, 2013; Stacey, 2011; Turner, 2012). To be mathematically literate, a person should be able to perform well a whole process of modeling from the first step investigating relevant information from real world situations within the process of *formulating* problems mathematically, to the next steps of mathematical modeling, *employing* mathematical procedures in order to obtain a mathematical results which is then applied to generate an *interpretation* to answer a task.

However, Indonesian students' performance in the PISA mathematics survey has not shown satisfactory results as indicated by the country's rank, which was lower than others and its scores lower than the average PISA scores over the years 2003-2012 (Kemdikbud 2013; National Center for Education Statistics 2013; OECD 2013; Stacey 2011). These facts point out that Indonesian students have not been able to successfully execute mathematical processes (formulate, employ, interpret) to solve contextual mathematics problem as well as bring out Fundamental Mathematical Capabilities (FMC), a set of mathematical competencies needed to be activated in each of the mathematical processes mentioned in the PISA framework, namely reasoning and argumentation, communication, problem solving strategies, mathematising, representation, and using formal/symbolic language (OECD, 2013). Considering these facts, the need for PISA-like tasks promoting students' mathematical literacy is important to be developed as learning resources. This study aims to develop a set of PISA-like mathematics tasks that are valid and practical, and to explain the process of developing those tasks. Using the tasks, we will also describe the extent to which students perform mathematical processes on the tasks developed to get insights on why Indonesian students failed in mathematical literacy-based tasks such as PISA.

PISA-like mathematics tasks

We call the tasks developed in this study as PISA-like mathematics tasks since their characteristic satisfies the framework of PISA mathematics problems released by OECD (see http://www.oecd.org/pisa/pisaproducts/). We tried to use the PISA

mathematics framework 2012 as a primary source in developing the tasks. Some previous studies that developed PISA-like tasks were conducted with a variety of purposes. For instance, Edo, Hartono, and Putri (2013) developed PISA-like tasks on measuring students' modeling abilities. Meanwhile, Lutfianto., Zulkardi, and Hartono (2013) developed tasks based on context category. In this study we developed PISA-like tasks by looking at the task design mentioned in PISA framework 2012.

Method

Developing PISA-like tasks

In developing tasks we used the framework's 'formative evaluation' as shown in Figure 1.

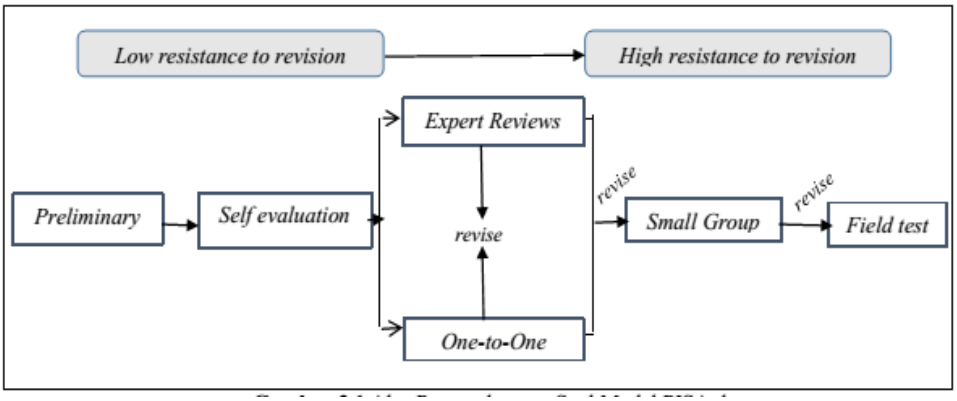


Figure 1. Formative evaluation (Tessmer, 1993; Zulkardi, 2002)

The development process started with the *preliminary* step of grasping the idea of mathematical literacy obtained from literature and designing an initial prototype comprising 9 tasks grouped as content, context, or process. This prototype, which was *self-evaluated* became *prototype 1*. In *expert review*, we involved eight experts to validate the tasks in terms of content, construct, and language: two experts from the *Mathematics Expert Group* PISA team (Prof. Kaye Stacey and Dr. Ross Turner) in Australia, five PMRI experts (PMRI: Indonesian version of Realistic Mathematics Education), one PISA Indonesian researcher, and one mathematics teacher. Simultaneously with expert review, five students in *one-to-one validation* evaluate particularly on how they understand information, e.g. picture, phrase, table within the tasks. All students who were involved in this study were around 15 years old the age group targeted by PISA (Stacey, 2011). These results gave important suggestions to revise the tasks to become *prototype 2* so that those could be re-evaluated in a *small group* of twelve students. In this step, we firstly obtained students' performance in solving the tasks by scoring and analyzing a variety of students' answers. We used this data with a view to assess students' real performance in larger tests, i.e. the *field test* which was then conducted after the small group test. The field test was conducted on fifty students aged fifteen years old of heterogeneous background (sex and mathematical ability) from senior high school SMAN 1 Palembang. From this development process, we finally obtained a set of PISA-like tasks as *prototype 3* which were used as instruments for investigating students' mathematical literacy

Investigating students' profile of mathematical literacy

To investigate students' errors, we used the framework's 'source of error in mathematical literacy examination' adapted from Newman analysis (Clements, 1980), PISA governing Board (2013) and Vale, P., Murray, S., & Brown, B. (2012) as shown in the following diagram.

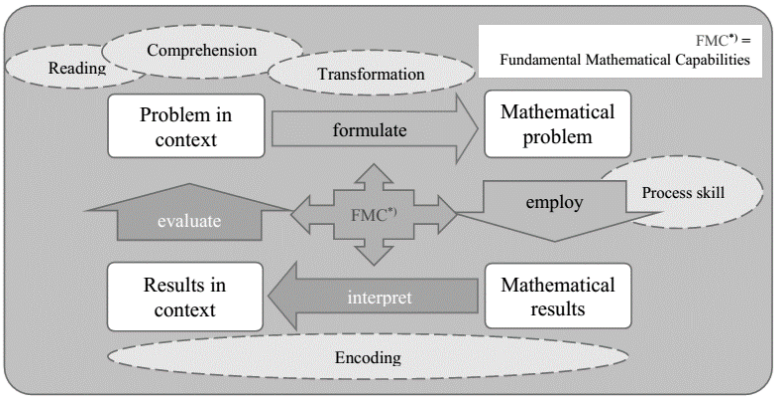


Figure 2. Source of error in mathematical literacy examination (Adapted from Vale et al, 2012 & PISA Governing Board OECD, 2013)

In the field test, we interviewed eight out of fifty student participants to gain insights on how they performed on the mathematical literacy tasks. Table 1 shows the description of each of possible errors of students.

Table 1 Source of error in mathematical process

Mathematical process	**Error Code**	**Description**
Formulate	R-error (*reading error*)	Students do not understand what is known, i.e. fail in identifying or making sense with certain word/single term, or pictorial and textual information; do not read general information given preceding the item; or do not provide any written answers.
	C-error (*comprehension error*)	Students understand what is known but do not understand what is asked because they do not understand the meaning of overall relationship of the information given.
	T-error (*transformation error*)	Students can identify what is known, what is asked, concept related to the item. However, they are unable to synthesize it into a corresponding mathematical models. This is because the students do not yet have an appropriate problem solving scheme of the solution.
Employ	P-error (*process skill error*)	Students are able to make an appropriate mathematical model, but fail in operating

Mathematical process	Error Code	Description
		symbolic/algebraic/procedures working within formal mathematics itself. As a result, they get wrong mathematical results.
Interpret/ Evaluate	E-error (*encoding error*)	Students are able to determine the precise mathematical results, but fail in writing a final answer (e.g. choose wrong answers on multiple choice questions). This is because it does not fit with the context of the given problem.

Adapted from Newman error analysis and source of mathematical literacy examinations (Clements, 1980; Vale, *et al*, 2012)

Results

Final revised tasks

In developing the tasks, we underwent a validity and practicality process. Validity of the tasks was confirmed mainly from the process of *expert review* from which recommendations on the tasks were made. Here, we applied the experts' suggestion to revise the tasks. Meanwhile, practicality of the tasks was confirmed not only from the experts but also from the students involved in *one-to-one* and *small group* who might have clearly understood the tasks or were ambiguous about them. Table 2 shows the profile of nine tasks on protoype 3 categorized as content, context, and process.

Students' mathematical literacy profile on the tasks

Table 3 shows the levels at which students' errors occurred in each task. Based on the percentages in Table 3, majority of the students' failure involved the process of 'formulate', i.e. about 52.9% or 38 out of 72 item. Comparing among the three processes within the 'formulate' process, transformation is the most frequently observed process error compared to the other two processes, i.e. reading and comprehension. Thus, we can say that most students had difficulty in providing mathematical structure to the problem presented in some contextualised form. Table 3 also shows only the small percentage, i.e. 31.7% of students' obtained the full score. This indicates the students' low ability in performing a mathematical process.

Table 2 Profile of PISA-like tasks

Task	Context	Content				Process			Description
		CR	SS	Q	UD	F	E	I	
1	Societal				√	√			Interpret statistical information about percentage of faulty assembled phones between two companies
2	Personal				√			√	Interpret information about a score of a match between two futsal teams in a tournament which is implicitly stated from information given in a table
3	Personal			√		√			Analyze hidden information about futsal scoring based on data presented
4	Scientific		√				√		Calculate surface area of infiltration area of a biopori hole
5	Scientific	√						√	Determine an appropriate graph representing a certain trend describing relationship between radius length of base on a biopori hole and maximum amount of waste that can be loaded in a biopori hole
6	Scientific		√			√			Determine maximum number of biopori if the measure of a field is given
7	Societal			√				√	Determine the best design of a temple representing a given photograph by finding the best approximation of the number of small stupas on the temple
8	Societal	√				√			Determine the number of temples which should be restored based on the pattern given
9	Scientific	√					√		Calculate weight loss needed to reach a certain BMI *(Body Mass Index)*

Note: CR=Change and Relationship; SS=Space and Shape; Q=Quantitiy; UD=Uncertainty and Data; F=Formulate; E=Employ; I=Interpret

Table 3 Students' level of errors in performing mathematical processes

Problem	Formulate			Employ	Interpret	
	I	II	III	IV	V	VI
1	2	1	0	1	2	2
2	0	0	5	0	0	3
3	3	1	2	0	0	2
4	2	3	2	0	0	1
5	0	0	3	0	4	1
6	2	0	3	1	0	2
7	1	0	3	1	0	3
8	0	0	3	1	0	4
9	0	0	2	0	1	4
Percentage	13.9%	7.0%	32.0%	5.6%	9.7%	31.7%

Note:
I= reading error, II= comprehension error, III=transformation error, IV= process skill error, V=encoding error, VI=complete mathematical process (full score)

$$\text{Percentage} = \frac{\text{the number of error items based on the analysis}}{\text{the total number of analyzed items (72)}} \times 100\%$$

To determine students' mathematical literacy levels, we present some examples of their written work and interview data on Task No. 2.

FUTSAL TOURNAMENT

Source: modernfutsal.com

At the end of this year, a subdistrict government held a futsal tournament involving some futsal teams from schools within the subdistrict.

Task No. 2

To prepare for the tournaments, three futsal teams from Al-Hikmah school held a futsal training in their schoolyard once a week. Each of them faced each other in a match exactly once. The following table presents the results of the training this week.

Team	Won	Lost	Drawn	Goals	
				Goals For	Goals Againts
Putra FC	0	2	0	3	7
Rajawali Club	1	0	1	5	3

Garuda United	1	0	1	6	4

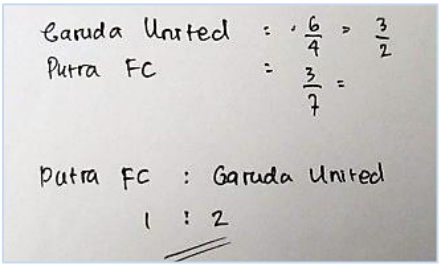
Note:
Won :The number of matches won
Lost :The number of matches lost
Drawn :The number of matches
Goals for: The number of goals scored
Goals against: The number of goals conceded

What is the score of the match Garuda United vs Putra FC? Explain your reason.

The context of a futsal tournament is often encountered by students because it is closely related to their daily sport activities. Therefore, this task is categorized as a *personal* context. The task assesses students' reasoning in interpreting the information presented in the table. The process of interpreting seemed most dominant in the completion stages of mathematical literacy because the need to evaluate relationships among information that are clearly presented in the table involves quite high FMC activation. To that end, this task is categorized in the process as "*interpret"*

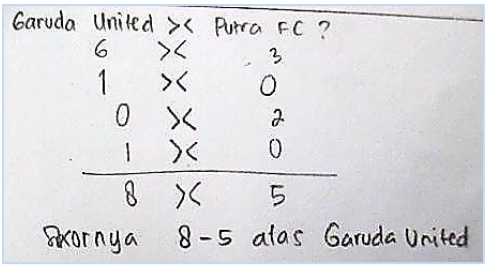
The analysis of students' written work showed that there were only 20% of students who answered this task perfectly. Common mistakes occured when students provided logical arguments to support their conclusion in determining the score of the match. The results of the interview showed that there were only three students who answered correctly, two students who failed in comprehension, while the other 3 failed in transformation. Here are examples of student's works demonstrating this situation.

S-1



Translation:
Garuda United =
Putra FC =
Putra FC:Garuda United 1:2

S-2



Translation:
The Score is 8-5 (Garuda United won)

Figure 3. Students' work on task no 2 (S-1 and S-2)

S-1 wrote the score of the match as 1: 2 with no logical reason that supports the answer, even though he seemed to try to make a ratio between goals againts and goals for the Rajawali united team. Likewise, S-2 failed during the stage of understanding the

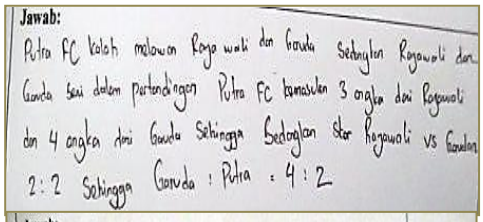
problem on the task. Here is an excerpt of the interview with S-2 which indicates the student's error in comprehension (C-*error*)

Interviewer	: Please read the task? Then reveal what the task asked
S-2	: What is score of Garuda team againts Putra FC.
Interviewer	: Do you know what to do?
S-2	: I think it's about ratio presented in a table
Interviewer	: Is there any word/ phrase/ information that you do not understand?
S-2	: Obviously understand
Interviewer	: Ok, now what do you do to solve this problem?
S-2	: The task ask to find score, doesn't it? So, on match Rajawali club vs Putra FC, I wrote 6 ><3 for goals for, then 1><0 for drawn game, 0><2 for lost game, and 1><0 for won game. Then I add up those to get 8-5. *(C-error)*
Interviewer	: Why did you add up?
S-2	: No reason, just guess it

In the interview, S-2 appeared to not have fully understood the information in the table, such as the meaning of goals for and goals against. Consequently, he understood inappropriately what the task really asked, which was to perform transformation during the process of formulating the problem. Hence, he failed to build mathematical arguments to support his strategy of adding up the numeric values within the columns of the table to determine the score of Garuda and Putra FC. Thus, we coded this error as *comprehension* error. Comparing the work of S-2 with students who answered correctly, the following are the results of the work by S-3 and S-4.

Translation
Putra FC lose in the match, Rajawali and Garuda make drawn match. Putra FC has 3 goals againts from Rajawali club and 4 goals againts from Garuda. Meanwhile, score Rajawali vs Garuda is 2:2. So

S-3



Translation
Putra FC vs Garuda United
Rajawali Club = 1-1
Garuda United = 1-1
So, Putra FC vs Garuda United 2:4

S-4

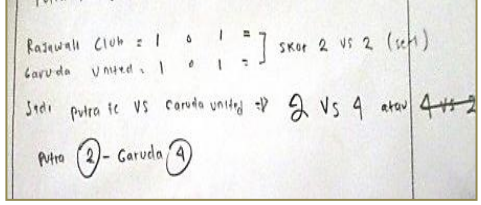


Figure 4 .Students work on task no 3(S-3 and S-4)

Based on the work of S-3 and S-4, it can be concluded that they successfully completed all the mathematical literacy processes from formulating the problem to interpreting results back to the initial problem. The process of *formulating* was performed by identifying any information from tables such as meaning of goals for-goals against before establishing a relationship among the information in the table. In

transformation, they tried to make the first step of solving strategy by identifying a key idea, namely finding the score of the drawn match. In *employing* mathematical procedures, they calculated the goal distribution of each team. In the end, they *interpreted* properly by concluding that the score of the match GU vs Putra FC is 4-2.

Discussion and Conclusion

Using the framework of formative evaluation, we gained 9 valid and practical PISA-like tasks distributed among three domains. We then used those tasks to investigate how students performed mathematical literacy processes.

To sum up, we argue that Indonesian students in this study have difficulties in completing mathematical literacy processes, mainly in formulating problem mathematically. This finding is closely related to students' performance who found difficulties to activate FMC in demonstrating the three mathematical processes: formulate, employ, interpret. The results provide a basis for further research into the possible causes of students' difficulties in solving mathematical literay-based tasks like PISA tasks. Teachers and researchers can use the profile of students' mathematical literacy explained in this study as a reference in designing mathematical literacy based learning as well as a diagnostic tool in evaluating students' learning outcomes.

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