The role of error analysis in problem-solving lessons to improve the perception of schoolteachers about the assessment

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Introduction

A research project - PROF-OBMEP (Baldin et al., 2013) aims at the improvement of teaching and learning mathematics for schoolteachers of 6th to 9th grades (11 to 14 years old) of Basic School System in Brazil. It has been carried out in recent years based on the theoretical framework of the content knowledge for teachers defined in (Shulman, 1986), as described in another paper of this conference (Baldin, 2015). The present paper follows the concept of pedagogical content knowledge under the perspective of the connection of research on teaching and learning with the cognitive research through studies of "students' misconception and influences to subsequent learning" (Shulman, 1986, p.10).

The main purpose of this article is to discuss the role of error analysis to improve the perception of schoolteachers about assessment through the activities of problem solving proposed as "learning task for teachers - PLT" (Silver, 2009) in the project PROF-OBMEP. One remarkable outcome of the project is precisely the desired change of perception of schoolteachers when they realize the potential to improve their teaching skills and the learning of their students as they can understand the diverse nature of errors and misconceptions that influence the achievement of the knowledge. This feature of assessment promoted by quality assessment is of utmost importance to plan the next lessons and elevate the quality of learning of school mathematics, constituting one of the motivations of the project PROF-OBMEP in working out Lesson Study principles in Brazilian schools. Authors as in (Becker & Shimada, 1997) have already pointed out the potential of problem-solving approach to teaching by Lesson Study methodology, as the strategy to promote mathematical thinking (Isoda et al., 2007; Isoda & Katagiri, 2013).

The error analysis of the project PROF-OBMEP

The project PROF-OBMEP proposes to in-service teachers the experience of the resolution process of problem solving, in order to understand the difficulties of their students according to the school grades and the connection of mathematics concepts of problems with the school curriculum. This approach is aligned to the research topic PBPD - Practice Based Professional Development (Ball & Bass, 2003; Silver, 2009).

The problem-solving lessons are a challenge to schoolteachers in Brazil because these require teachers' competencies to conduct meaningful inquiries during the classroom time to promote active learning of students, who in this way can experience the efficacy of one's own solving strategies subject to errors and disclosure of misconceptions, as well as failures in technical procedures. This feature of the problem-solving steps are clearly an asset to deepen the meaning of assessment that can go beyond the simple act of correcting the answers as "right" or "wrong."

Good closed-ended problems are usually present in textbooks, but the open-ended problem approach permits new perspectives in teaching mathematics, as they "promote implications for improving assessment in problem solving…higher-order thinking skills in school mathematics" (Becker & Shimada, 1997). Countries like Thailand (Inprasitha, 1997;

Inprasitha, 2006) and Singapore (Ginsburg et al., 2005) that included the open-ended problems as a central theme of problem-solving lessons in the modernization process of the curriculum have experienced the power of demystification of mathematics as “wrong x correct” science, thus opening a new perspective for investigation and discovery in learning process through problem-solving approach.

In Brazil, although the National Standards consider as the core activity of school curriculum (Brasil, 1998), schoolteachers usually conceive a problem-solving lesson as an application of techniques and procedures that are transmitted orally right before the proposition of some standardized problems of textbooks. The drilling type exercises are often considered problem-solving lessons. This causes weak preparation, in general, of Brazilian students in solving word problems, the true problems that require reading, interpreting the data, finding the question to solve, remembering from one’s knowledge the concepts and procedures to figure out a strategy to attack the problem and being able to check if the result achieved in this way is really meaningful to the context.

Being able to justify the strategy and the procedures by means of conceptual knowledge is the key step to have success in other problems. The steps of many problem-solving methodologies are disseminated among problem-solvers, the most familiar being Polya’s (Polya, 1945). However, the limited understanding of schoolteachers about the didactical significance of problem-solving lessons becomes one of the causes of poor performance of Brazilian students in large-scale comparative assessments such as the OECD – PISA (57th out of 64 countries in 2011).

The project PROF-OBMEP introduces to teachers a “professional learning task – PLT” (Silver, 2009), which works out transformations of classroom dynamics into a problem-solving lesson providing space and time to the students to solve a problem, executing by themselves each phase of problem solving. The change from teacher-centered explanation to student-centered classroom activity is quite innovative for Brazilian schoolteachers. The PLT of the project is a real exercise for teachers to learn how to teach, to make meaningful and stimulating inquiries, to foresee the students’ reactions, to be prepared for questioning during the lesson. The opportunity to listen to the questions asked by students is a gateway to better assessment, since the richness of the possibilities permit different perceptions of the level of the learning and the actual individual difficulties. A research on the impact of this change of paradigm of classroom dynamics on the comparative external assessment of students’ achievements is currently on the way.

Besides the study of the process of problem-solving that includes the revision of mathematics concepts and techniques, the innovative exercise of the PLT of the project is to work out the ability to recognize and explore different errors that might come up during the lessons, as students present their answers with authorship of strategies and techniques, justifications as well. When it is the teacher who explains a solving strategy, confident of his (her) own knowledge, the general attitude in the evaluation activity is to consider ‘wrong’ an answer different from his (her) own without any questioning, and no chances for open-ended problems. On the other hand, a lazy student usually tries to copy and imitate the teacher’s solution without developing critical thinking as well as to seek approval from the teacher of his (her) answers before validating or investigating the meaning of one’s result. Moreover, the reproduction in the assessment tests of traditional and familiar exercises drilled in trainings may undermine the results of the evaluations, not stimulating the smart students to broaden their learning skills. In the preface of Isoda & Katagiri, (2012), Isoda highlights the role of problem-solving approach in Japanese lessons as the strategy to develop mathematical thinking.

Therefore, the step of validation and investigation performed by students becomes a source of investigation for teachers in their task of learning to assess. This is a paradigm shift in the assessment attitude of teachers aimed by the project.

Another important role of assessment through problem-solving is that the identification of students' misconceptions helps teachers to improve the lesson planning, elevating it to the next level, motivating the research spirit for new teaching materials and methodologies. This is part of the strategy to introduce Lesson Study activities into Professional Development projects in Brazil. In this regard, the continuity of the project has aimed at stepping forward the initial understanding of teachers about the problem-solving approach into the next level, that is, the opportunity of exploring and discovering the diverse facets and strategies of solving a problem has allowed conjecturing new problems and extended concepts and contexts, as it happens in open-ended problem approach.

The project also stimulates the integration of ICT as teaching and learning tools to motivate the mathematical thinking, taking along the advantages of the readiness of students, especially in the transition years of 6th and 7th grades, in manipulating concrete objects to simulate the problem and helping to bridge from the previous knowledge to abstract representation the language of mathematics (Baldin et al., 2013).

Illustrative examples of students' errors: Case of fractions

Brazilian teachers, especially of elementary grades, tend to teach by repetition of procedures and mechanical algorithms without meaningful explanations. The education system of teachers of these grades forms generalists with much pedagogical theories and little specific content knowledge.

In Brazil, 6th grade is crucial because it corresponds to the transition from elementary first cycle (1 to 5th grades), taught by teachers with weak content knowledge in mathematics, to second cycle (6 to 9th grades) taught by teachers educated in universities. In the Teacher Education Courses at universities, the mathematics emphasis is on higher mathematics, and the pedagogy is unsatisfactory in making connections to the specific content of school curriculum.

The following example was taken from a research project of a graduate student to detect misconceptions of previous knowledge and operating abilities in his 6th grade classroom. (Baldin & Carrijo, 2013). His project has been carried out as part of Lesson Study project in Brazil. The research helped him to plan the next adapted lessons to the reality of his students' difficulties as well as provided resources to the project PROF-OBMEP.

First, exercises were proposed to diagnose the previous knowledge and those resulted in finding surprising errors that confirmed serious problems in the teaching of fractions at elementary level, and challenged the teacher to plan different approaches to diminish the gap. Therefore a teacher can learn how to teach better through analysis and interpretation of errors.

Example: What fraction of the big rectangle represents the colored parts?

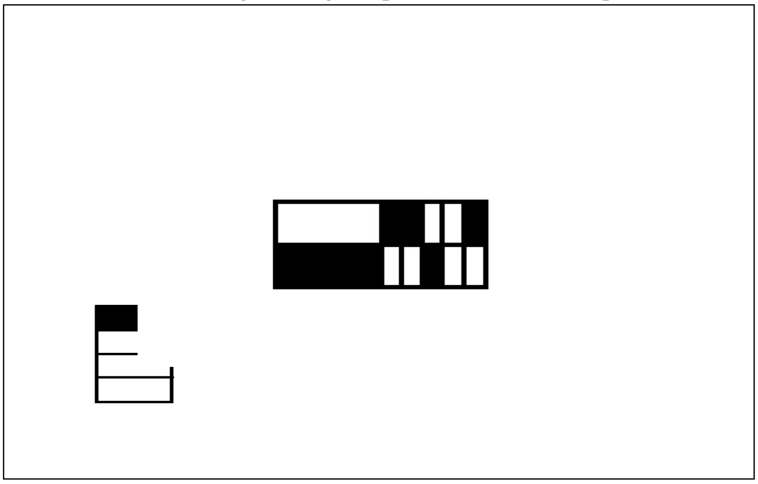

Figure 1: Identifying the concept of fractions in part-whole pictorial representation

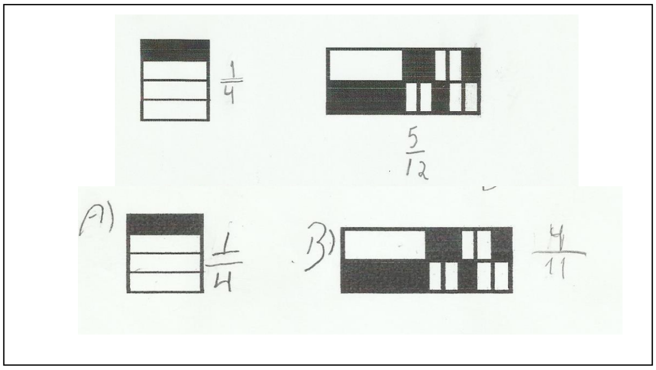

Figure 2: Some frequently detected misconceptions

The participative learning environment provided by the teacher to stimulate the dialogue and questioning, giving opportunity to the student to justify his answers expressing his own thoughts, clarified the true nature of errors in Figure 2. The understanding of the basic concept that underlines the fraction representation, “the division (fraction) of a whole into equal parts” was lacking in the perception of the students. The error can easily be fixed explaining the need for the equality of sizes of the pieces. Nevertheless this is very serious evidence that fraction is a meaningless concept for the students, since the answers were based only on mechanical memorization of some fact.

In the new dynamics of the classroom, as the student learns the correct concept, the teacher learns as well to diagnose the nature of difficulties, stepping forward in his conception of assessment. Previous to the project, the teacher’s task was simply to grade the answers and register the statistics table with students’ achievements, but with the new dynamics the teacher becomes a researcher and better educator.

In the next example a misconception error caused by another factor is analyzed, demanding new teaching strategies (Baldin & Carrijo, 2013). In this reference, the same example is analyzed from the perspective of Lesson Study methodology, whereas in the present paper we focus the assessment point of view.

Example: Represent the indicated fraction in each figure.

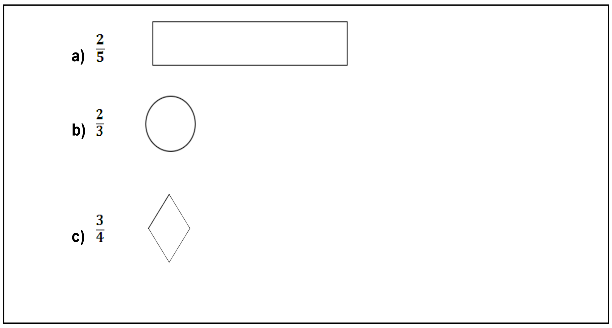

Figure 3: Exercise of conceptual knowledge of fraction

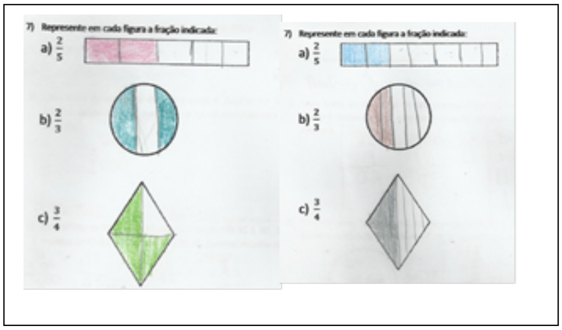

Figure 4: Misconceptions requiring different interpretation

The analysis of the errors shown in Figure 4 is a confirmation of the importance of the pedagogical content knowledge of schoolteachers about specific content knowledge of basic mathematics and of teaching methodologies. The interpretation of the errors has led to findings about the weakness of basic knowledge of school mathematics of teachers of elementary schools in Brazil, in particular of the curriculum of 3rd to 5th grades, when the topic of fractions turns out to be a major difficulty.

The answer on the left side of Figure 4 presents a procedure induced by examples, that is, the use of vertical lines to divide a circle into three stripes, induced by the rectangular bar above it. The division of the rhombus is a familiar illustration of textbooks, while the answer on the right side presents still the idea of vertical stripes. The intriguing error on the total number of stripes on the right is a result of amazing teaching flaw of the teacher who does not know the actual meaning of mathematics activities in a school curriculum. The number of pieces drawn is “the sum of the numbers, that is, 2/5 means you should add 2 + 5 = 7, and then, color two of them.” This interpretation is a consequence of naïve belief that teaching mathematics means teaching calculations. Therefore, every number in the text of a problem must be operated, addition being the easiest choice. However strange this assertion sounds, the reality of Brazilian schools is plenty of this misunderstanding about numbers and arithmetic. Therefore problem-solving lessons, especially of geometry without measures (numbers) is another theme of research in Professional Development courses, especially in the project PROF-OBMEP.

A careful analysis of the causes of such misconception made the teacher-researcher figure out that maybe the drawing pictures to represent the concept of fractions is still ahead of the level of mathematics abstraction of his students, even at 6th grade level (Carrijo, 2012). The teacher discovered then that the natural process of learning through visualization, manipulation of concrete objects and then proceed gradually to abstraction of representation language is known to curriculum developers; and the textbooks from Singapore or Japan, beside others, are evidences of the pedagogy of mathematics education (Fong & al., 2013; Gakkotosho, 2013).

The next orientation to remedy the situation was to use the material named "the fraction case," a manipulative to establish the concept of a "whole" as a fixed rectangular basis to be filled with stripes of different sizes, each color of same width, simulating the fraction units ½,1/3, ¼ … 1/12. The multiples of fractions are given by transparent screens of correspondent size to be put on the pieces of colored stripes and used for comparison between fractions. See the Figure 5 below.

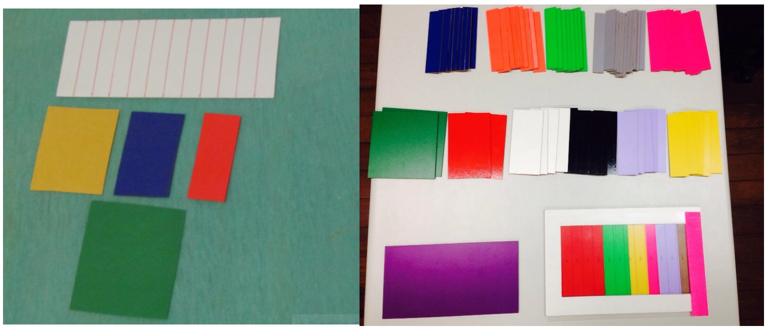

Figure 5: Fraction case, a manipulative to help children build the concept of fractions

The idea of using concrete material to fix the misconceptions works well, and this material is being used subsequently in different schools by other teachers, in different regions.

Regarding the assessment issue, the external comparative evaluation of student achievement carried by official educational departments usually include basic tests about the topic of fractions and its operations, as is required by the core curriculum of basic education. (Brasil, 1998) Therefore, a solid knowledge of fractions and their basic operations is fundamental. Moreover, the idea of comparing quantities and representations is basic in understanding the extension of ideas of mathematics, crucial also in the next level of rational numbers, when the competency to transit between different usages of these numbers is strongly required in algebra problems.

The "fraction case" has shown to be a useful asset to teach fractions for children with conceptual difficulties, because its manipulation is based on the comparison between pieces and a fixed "whole," impeding the imperfections of hand drawings that can lead to think by visualization and not by organized argumentations. A nice outcome is the possibility to work the meanings of addition and subtraction, even for negative rational numbers. After getting some confidence, students quickly abandon the material to start representing with geometric figures, when the teacher can combine the idea of fractions with geometrical properties of the models. See Figure 6.

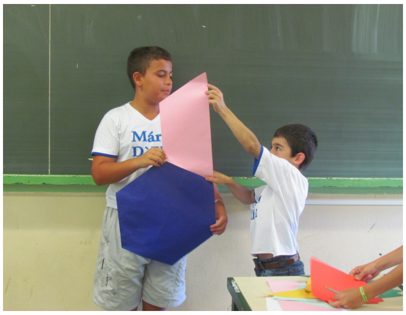

Figure 6: Learning geometry and fractions in a participative environment

The error analysis has shown to be very important also to interpret students' thoughts in word problems like the following, taken from the OBMEP - Brazilian Olympiad in Mathematics for Public Schools (http://www.obmep.org.br), a large competition meant to contribute to quality teaching in Brazilian public schools.

Example (adapted): A bowl with berries was left on the table with recommendation that the brothers A, B, C, D and E should divide equally among them. A was the first to wake up, took 1/5 of the berries and went away. B was the second to wake up, did not realize that A had already taken his share, and took 1/5 of the bowl he found and went away. The rest of brothers, C, D and E woke up together and divided equally the rest they found. The questions are: a) What fraction of the berries was the share of B? b) Who took more and who took less? c) If we know that nobody took more than 20 berries how many berries were there at all?

The mathematics concept knowledge to be assessed in this problem is simply "fraction, its meaning, comparison of fractions and operations," so it is very well suited to evaluate the reading ability of 6th grade students.

This problem was considered extremely difficult by teachers because it does not require doing complex calculations, but it demands understanding the problem and the decision of what operations must be performed as the central part of resolution.

Some surprising answers have appeared disclosing the knowledge children bring from elementary school. When the previous knowledge is faulty, teachers have the mission to help fix the difficulty. Without the perspective of investigation of learning difficulties in assessment process, a teacher would be restricted to remain only a grader of tests, not contributing to quality learning of students.

Figure 7 presents a very frequent answer proposed to teachers to analyze as a task in the project.

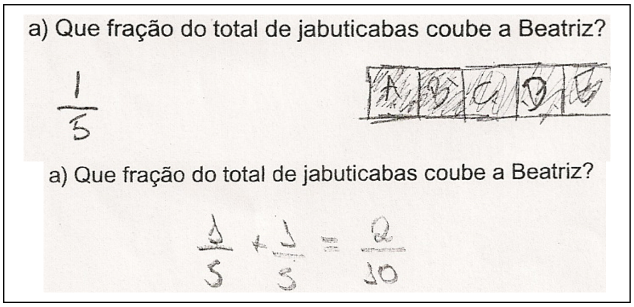

Figure 7: The most frequent answer: 1/5. Why?

In Figure 7 we find the frequent wrong answer as well as a classic error encountered around the world about the procedure in an operation of addition. It is a task for teachers to analyze the type of argumentation behind these answers. It is quite interesting that when one reads the text loudly stressing that B is the second to take the berries and B is one of 5 brothers, the justification of students begins to come up.

It is an important task of teachers not to judge the answer simply as wrong, but also to discover the rationale underneath the answer. It is surprising that the social justice background that implies the equal sharing among brothers is one of interesting results learned by researchers through the analysis of the students' justification. Now, will a teacher be able to analyze the reason of addition in the second answer? In Brazil, a teacher traditionally would mark it as wrong answer and would try to teach the procedure to it as a solution to this behavior, not paying attention to students' point of view.

When the student is allowed to explain his (her) reasoning, some interesting answers came out, for instance, such as B was the "second" to take the berries, therefore the number 2 in the answer. This kind of perception defies the schoolteacher to seek alternative teaching strategies to foster meaningful lesson planning and further learning of correct concepts.

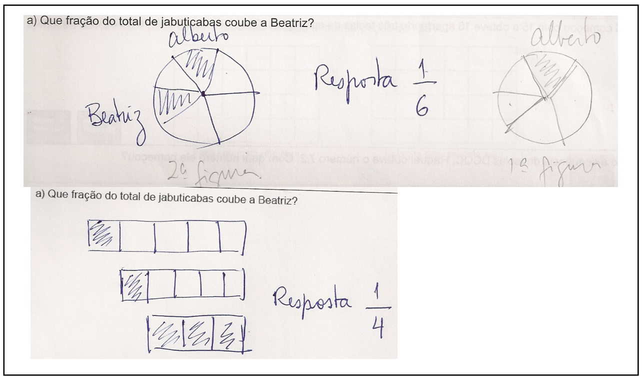

Figure 8: What is wrong here? Are the answers totally wrong?

Figure 8 requires a different kind of attention from the teacher. If there were a test to check the correct alternative, certainly the students would have failed in this item. However, the opportunity to analyze these answers, trying to understand from the students' perspective, allows the teacher to realize that the mathematical thinking is perfect in both answers; it is the strategy of representing the picture model of the problem that is mistaken, which leads to final wrong answer. It is clear that the two steps of problem solving, including the most difficult part of the comprehension of text are accomplished. To help the student to understand what the pitfall was, an experiment with concrete material of fraction case to follow the reasoning could be a valued teaching strategy.

When the teacher masters the skills of conducting a lesson through inquiries and discoveries, the assessment activity gains a new dimension entailing a more effective lesson planning, as it is conceived in the cycles of Lesson Study as well as opening the space for open-ended problem approach upon the familiar textbook problems.

Conclusion

The project PROF-OBMEP is a professional development course that proposes to in-service teachers the experience of the resolution process of problem solving, in order to understand the difficulties of their students according to the school grades, as well as the connection of mathematics concepts of problems with the school curriculum.

The project explores the problem-solving approach to develop the competency of in-service teachers in teaching skills and becoming better educators by learning the potential of quality assessment to help the students.

The examples discussed in this paper show that through the step of validation and investigation of problem-solving methodology, the activity of assessment, traditionally restricted to grade the answers and classify the students in a ranking of achievement, gains a new importance as it plays a definite role in improving the teacher's perception in understanding the misconceptions about mathematics concepts, and as sources for research of adequate teaching materials and strategies.

The above-described feature of assessment in problem-solving lessons distinguishes it also as a PLT- Professional Learning Task, so contributing to the research on Teacher Education as indicated by Silver (2009).

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