

# **Profiling teacher capacity in statistical thinking: A comparative study between China and Australia**

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## **Introduction**

In the official curriculum documents of many countries, statistics and statistical reasoning have become part of the mainstream school curriculum. In The Australian Curriculum: Mathematics (ACARA, 2010), “Statistics and Probability” is one of three key content areas. In its overview statement to this strand, ACARA (2010) states that: “Statistics and probability initially develop in parallel, and the curriculum then progressively builds the links between them. Students recognise and analyse data and draw inferences ... they develop ... to critically evaluate chance and data concepts ... and develop intuitions about data” (p. 2). A corresponding strand, Chance and Data has been present, for at least five years, in related state curriculum documents, e.g. VELS (VCAA, 2008) and the Mathematics Developmental Continuum (DEECD, 2006). China’s newly revised Mathematics Curriculum Standard for Compulsory Education (Ministry of Education, 2011) also presents a single strand entitled Statistics and Probability. In the overall objective for this content strand, it is stated that “to experience the process of collecting and dealing with data in practical problems, as well as using data to analyse questions and obtaining information” in “knowledge and skills” (p. 8); and it refers to “to experience the significance of statistical methods, to develop ideas of statistical analysis and to experience random phenomena” in “mathematical thinking” (p. 9). As for Philippines, the contents of mathematics curriculum include Numbers and Number Sense, Measurement, Geometry, Patterns and Algebra, and Statistics and Probability (Department of Education of Republic of Philippines, 2012, p. 2). Statistics and Probability as a strand is all about “developing skills in collecting and organizing data using charts, tables and graphs, understanding, analyzing and interpreting data, dealing with uncertainty and making predictions and outcomes.” (p. 5).

These intentions are endorsed by Garfield and Ben-Zvi (2008) who point out that in contrast to traditional approaches to teaching, which focus on computations of theoretical probability, new emphases are squarely focused on understanding data and development of statistical thinking and literacy (p. 7). They argue that “the goals for students at the elementary and secondary level tend to focus more on conceptual understanding and attainment of statistical literacy and thinking and less on learning a separate set of tools and procedures” (p. 14). These goals are reflected in the National Curriculum in Australia and China, where students are expected to learn and understand that: (1) explanations supported by data are more powerful than personal opinions or anecdotes; (2) variability is natural and is also predictable and quantifiable; (3) association is not the same as causation, and; (4) random sampling allows results of surveys and experiments to be extended to the population from which the sample was taken (cf. Garfield and Ben-Zvi, 2008, p. 15).

However, the implementation of curriculum change is never simply from the top-down. Teachers’ interpretations and responses at the level of practice are never simple

reflections of what is contained in official curriculum documents. While curriculum documents set out broad directions for change, any successful implementation of these “big ideas” depends on teachers’ capacity to apply subtle interpretations and careful local adaptations (Datnow and Castellano, 2001). We argue that teacher capacity is a key dimension in realising that goal.

### **Teacher capacity and mathematical knowledge for teaching**

While the term “teacher capacity” is not widely used in mathematics education research, it has clear connections with the research of “Pedagogical Content Knowledge” by Shulman (1986; 1987) and “Mathematical Knowledge for Teaching” by Ball, Thames & Phelps (2008).

#### *Shulman’s model*

Shulman (1987) identified pedagogical content knowledge as the category most likely to distinguish the understanding of the content specialist from that of the expert teacher. The importance given to PCK suggests that what is needed in mathematics teaching is not just knowledge of the subject, or general knowledge of pedagogy, but rather a combination of both. However, after twenty five years of exposure to Shulman’s thinking, Petrou and Goulding (2011) conclude that: “Although Shulman’s work was groundbreaking and his ideas continue to influence the majority of research in the field, later researchers in the same tradition argue that it is not sufficiently developed to be operationalised in research on teacher knowledge and teacher education” (p. 12). We note that Shulman did not write specifically for mathematics teaching, but for all teaching subjects; and that his categories tend to reflect the educational context of the USA where there was no national curriculum.

#### *Michigan model*

Ball et al. (2008), while sympathetic to Shulman, prefer to use the term Mathematical knowledge for teaching (MKT). Within this idea, they identify four constituent domains or categories: (1) Common content knowledge (CCK) defined as the mathematical knowledge and skill used in settings other than teaching; (2) Specialized content knowledge (SCK) as the mathematical knowledge and skill unique to teaching specific topics; (3) Knowledge of content and students (KCS) defined as knowledge that combines knowing about students and knowing about mathematics; and (4) Knowledge of content and teaching (KCT), which combines knowing about teaching and knowing about mathematics.

Among these four domains discussed by Ball et al. (2008), CCK is a primary component of mathematical knowledge, and needs to be combined with a teacher’s SCK, the subject matter knowledge needed for teaching specific mathematics content or topics. KCS and KCT are both intended to describe distinct knowledge for teaching. However, “content” used in the four categories may refer to: today’s worksheet, or this year’s textbook, or what is contained in official curriculum documents. In this sense, KCT may not be too far removed from Shulman’s category of curriculum knowledge under which he includes teachers having a grasp of relevant materials and programs. While these knowledge domains are intended to anticipate classroom use, their instructional consequences are only implied. What is more, what appears to be a common feature of both Ball et al. (2008) and Shulman (1986; 1987) is an interpretation of “curriculum” and “curriculum knowledge,” which may be based too

closely on their USA experience where curriculum knowledge can be interpreted simply as “the particular grasp of the materials and programs that serve as ‘tools of the trade’ for teachers” (Shulman, 1987, p. 8), and Ball et al. (2008) do not seem to have moved beyond this.

#### *Limitations of research on PCK and MKT*

Ruthven (2011) has presented four distinct conceptualisations of Mathematical knowledge for teaching – Subject knowledge differentiated; Subject knowledge contextualised; Subject knowledge interactivated and Subject knowledge mathematised – each of which is intended to move forward debate about and research – but in different directions. These four lines of thinking show that Mathematical knowledge for teaching is no longer a single unified idea. Researchers also need to be aware of the limitations of some or all of these four approaches: (1) all four have a strong focus on how to improve pre-service teachers’ mathematical knowledge needed for their teaching in the future; (2) apart from the first framework adopted by Petrou and Goulding (2011), the other three do not appear to place a strong emphasis on the way in which official mathematics curriculum documents are intended to guide teaching in many countries; (3) apart from the first framework, the other three tend to view knowledge for teaching mathematics in general terms, rather than considering the specific areas of mathematical content important for curriculum reform; (4) all four theoretical frameworks are not easy to conceptualise into empirically conducted research. Our own position on Mathematical knowledge for teaching is closest to that of Petrou and Goulding (2011). We use this framework to inform our construct of teacher capacity, and to show where it differs from that of Ball et al. (2008), especially in its stronger links to research on curriculum reform and school change.

#### *Teacher capacity model*

The term “teacher capacity” comes out of the literature of school improvement, school leadership and system reform (McDiarmid, 2006; Fullan, 2010). When used in this context, teacher capacity usually relates to teachers’ ability to understand and act on the reforms that policy makers are seeking to implement (Spillane, 1999). It is close to our definition of teacher capacity as professionally informed judgement and disposition to act. Researchers such as Floden, Goetz and O’Day (1995) emphasise that teacher capacity is multidimensional and evolving. Firstly, they argue that teachers’ ability to assist students in learning is dependent on teachers’ own knowledge, which includes knowledge of the subject matter, knowledge of curriculum, knowledge about students and knowledge about general and subject-specific pedagogy; secondly, they argue that, while knowledge interacts with skills, there is a considerable gap between what teachers believe they should be doing in the classroom and their ability to teach in the desired ways; and thirdly, they point to the importance of dispositions, since enacting reform requires having the dispositions to meet new standards for student learning and to make the necessary changes in practice.

There are clear parallels here with Ball et al. (2008) who make the equally strong point that any definition of Mathematical knowledge for teaching (MKT) should begin with teaching, not teachers. Any such definition must be “concerned with the tasks involved in teaching and the mathematical demands of these tasks (our emphasis). Because teaching involves showing students how to solve problems, answering

students' questions, and checking students' work, it demands an understanding of the content of the school curriculum" (p. 395).

## **Methodology**

### *Theoretical framework*

Our construct of Teacher Capacity, as professionally informed judgements and dispositions to act, is intended to capture a common ground between movements for school system and curriculum reform and the construct of Mathematical knowledge for teaching elaborated by Ball et al. (2008). Four criteria inform our theoretical model.

Criterion A – Knowledge of mathematics – is intended to be applied to the tasks that the students have completed or are being asked to complete. Knowledge of Mathematics is intended to capture the key mathematical ideas for teaching specific content.

Criterion B – Interpretation of the intentions of the official mathematics curriculum – is concerned with how teachers relate what is mandated or recommended in official curriculum documents of China and Australia to what their students are being asked to learn. This Criterion differs from MKT (Ball et al., 2008) in giving a greater emphasis to official curriculum documents and teachers' willingness to use them in planning instruction.

Criterion C – Understanding of students' mathematical thinking – is directly concerned with teachers' capacity to interpret and differentiate between what students actually do (or did) and to anticipate what they are likely to do. It implies that teachers are able to recognize the typical errors that students make and what mathematical thinking led to these errors.

Consequently, Criterion D – Design of teaching – places a clear emphasis on teachers' capacity to design teaching in order to move students' thinking forward and to respond to specific examples of students' thinking in the light of official curriculum documents. Criterion D is intended to give greater emphasis to how teachers use their professionally informed judgement to design practical teaching on specific topics.

Among the four criteria above, criterion D is intended to give greater emphasis to how teachers use their professionally informed judgement to design practical teaching on specific topics. Our model of teacher capacity is shown in Figure 1 (Zhang & Stephens, 2013).

As shown in Figure 1, each of the four criteria is focused on teachers' professionally informed judgements and dispositions to act, distinguishing them from the four knowledge domains of Ball et al. (2008). Criterion D (design of teaching) is put forward as the central component in our model. As the instructional embodiment of teacher capacity, design of teaching rests on strong connections with the other three criteria. However, we also anticipate that, when separated from criterion D, the interrelationships between the other three criteria will not be as strong as their relationship to criterion D (design of teaching) (Zhang & Stephens, 2013). What is more, each criterion above was elaborated in terms of four specific indicators (see Table 1).

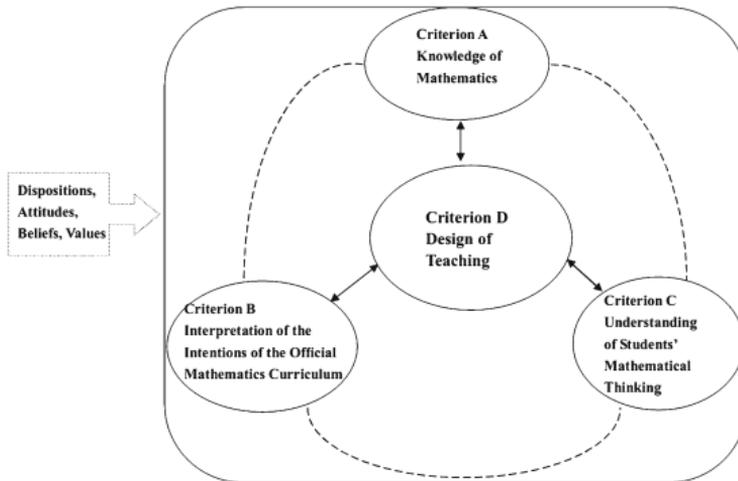


Figure 1: Model of Teacher Capacity

Table 1: Four criteria and associated indicators

Criterion A – Knowledge of Mathematics	Criterion C – Understanding of Students' Mathematical Thinking
<p>(1) Is the teacher able to solve the theoretical mathematical probability problem (Q1a) and be able to understand relationship between chance of real events and sample size (Q3)?</p> <p>(2) Does the teacher consistently understand that variability of theoretical probability always happens in natural events in real life (Q1b, 1d), and that variability has a certain range close to the theoretical probability (Q1c, 1e, 1f)?</p> <p>(3) Does the teacher understand the meaning of “variability” by giving specific certain information (Q2)?</p> <p>(4) Does the teacher recognize the difference between association and causation (Q4)?</p>	<p>(1) Is the teacher able to anticipate students' common difficulties and misconceptions on Question 1 (e) in questionnaire?</p> <p>(2) Does the teacher give clear and reasonable explanations to students' incorrect answers?</p> <p>(3) Is the teacher able to discriminate between students' different levels of understanding statistics and probability according to their answers, especially discriminating between incorrect answers?</p> <p>(4) Does the teacher recognize the importance of using familiar contexts, such as coin tossing or rolling dice, to help students understand the statistical features of (less familiar) situations that contain similar statistical characteristics.</p>
Criterion B – Interpretation of the Intentions of Official Mathematics Curriculum	Criterion D – Design of Teaching
	<p>(1) In design of teaching, does the teacher</p>

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| <p>(1) Does the teacher realize that “statistical thinking” should be valued in teaching and learning beyond the solutions of probability problems or does the teacher refer to relevant statements on statistical thinking in the official curriculum documents?</p> <p>(2) Does the teacher understand and support the intention of the curriculum of helping students understand key ideas of statistical thinking such as theoretical probability, sampling, sample size and inevitable variability in actual data, rather than calculating theoretical probabilities?</p> <p>(3) Does the teacher think it important to consider statistics and probability by linking natural events and real life?</p> <p>(4) Does the teacher show in his/her descriptions of developing students’ ability to read and understand data and information, which is important for their further learning and future life?</p> | <p>focus on the important key conceptions of statistical thinking (theoretical probability, sampling, sample size and inevitable variability in actual data, as well as using familiar contexts to simulate real world events), not focusing too much on general teaching strategies or overall descriptions on statistics and probability?</p> <p>(2) Does the teacher have the subsequent plan in next one or several lessons to respond students’ incorrect answers in Question 1 (e)?</p> <p>(3) Does the teacher have a longer-term plan to consistently develop students’ deep understanding of statistical thinking (see 1 above), not just aiming to have students correctly calculate theoretical probability problems?</p> <p>(4) Does the teacher, in his/her teaching, give concrete examples that are familiar and easy for students to understand to help them understand statistical thinking and its relationships with real life?</p> |
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*The research instrument*

Teachers were invited to complete a written questionnaire consisting of two parts. Part A has four questions which were based on tasks developed in previous research, containing some situations relating to statistical thinking that students are expected to meet. Question 1 was adapted from Shaughnessy et al. (2004):

A gumball machine has 100 gumballs in it. 20 are yellow, 30 are blue, and 50 are red. The gumballs are all mixed up inside the machine.

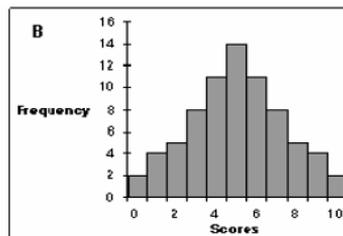
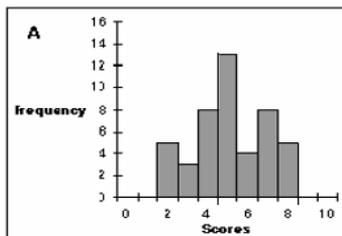
(a) Suppose you do the following experiment: you pick out a handful of 10 gumballs, count the reds and write down the number of red gumballs in one handful. How many reds do you expect to get?

(b) You replace the handful of 10 gumballs back in the machine and mix them up again. Now you draw another handful of 10 gumballs. Would you expect to get the same number of reds in every handful if you did this several times? Briefly describe why.

(c) How many reds would surprise you in a handful of ten? Why would that surprise you?

(d) If each time a handful

of 10 gumballs is taken, these are replaced and remixed before taking another handful



again, what do you think is likely to occur for the numbers of red gumballs that come out for a sequence of five handfuls? Please write the number of reds in each handful here.

(e) Look at these possibilities that some students have written down for the numbers they thought likely when they answered question d. Which one of these lists do you think best describes what is most likely to happen? Circle it. (A. 8,9,7,9,10; B. 3,7,5,8,5; C. 5,5,5,5,5; D. 2,4,3,4,3; E. 3,0,9,2,8; F. 7,7,7,7,7). Why do you think the list you chose best describes what is most likely to happen?

(f) In the above six repetitions of the experiment, what do you think will be the highest and lowest number of reds in one handful? Please discuss briefly why you think this.

Question 2 was adapted from Meletiou, and Lee (2002):

Look at the histogram of the two distributions on the right. Which of the two distributions you think has more variability? (a) Distribution A (b) Distribution B. Briefly describe why you think this.

Question 3 was adapted from Garfield, and Gal (1999):

Half of all newborns are girls and half are boys. Hospital A records an average of 50 births a day. Hospital B records an average of 10 births a day. On a particular day, which hospital is more likely to record 80% or more female births? (a) Hospital A (with 50 births a day); (b) Hospital B (with 10 births a day); (c) The two hospitals are equally likely to record such an event; (d) There is no basis for predicting which hospital would have that percentage of female births. Give a brief explanation of why you think like this.

Question 4 was adapted from Garfield, and Gal (1999):

For one month, 500 elementary students kept a daily record of the hours they spent watching television. The average number of hours per week spent watching television was 28. The researchers conducting the study also obtained report cards for each of the students. They found that the students who did well in school spent less time watching television than those students who did poorly. Which of the following statements is (are) correct?

- (a) The sample of 500 is too small to permit drawing conclusions;
- (b) If a student decreased the amount of time spent watching television, his or her performance in school would improve;
- (c) Even though students who did well watched less television, this doesn't necessarily mean that watching television hurts school performance;
- (d) One month is not a long enough period of time to estimate how many hours the students really spend watching television;
- (e) The research demonstrates that watching television causes poorer performance in school;
- (f) I don't agree with any of these statements.

For one statement that you agree with, explain why you think that way. For one statement that you disagree with, explain why you think that way.

Part B of the questionnaire had three questions which asked teachers to consider teaching implications arising from the questions in Part A. Specifically they were asked to consider common misunderstandings and difficulties for students in the Part A questions; how the key mathematical ideas or critical points presented in these questions

are addressed in their respective country's official curriculum documents, and; how to design some lessons to help students understand these key ideas.

#### *The participants*

There were 17 Australian secondary and primary schools randomly selected in both urban and rural regions in Melbourne. Up to four teachers of Year 6 or Year 7 in each participating school were invited to complete the questionnaire. The Australian sample consisted of 41 Australian teachers, 28 secondary teachers and 13 from primary schools. The China sample comprised 41 teachers randomly selected from training programs in Chongqing, Hangzhou and Wenzhou. Twenty-eight were secondary and 13 were primary teachers.

#### **Qualitative analysis**

The following examples provide evidence of Chinese and Australian teachers' (coded either as Teacher n CH or Teacher n AU) responses with respect to each of the four criteria. Teachers in both countries showed their different levels of understanding on all four criteria.

#### *Criterion A (Knowledge of mathematics)*

Teacher 57 AU responded to Question 3 of Part A: "Hospital B is more likely to record the 80% as it has a much smaller population... Larger samples or more trials give results that are closer to theoretical probability." This teacher clearly demonstrated understanding of the relationship between sample size and variation from theoretical probability.

However, Teacher 31 AU answered: "Both hospitals are equally likely to record 80% female births because the probability is the same for each birth to be a boy or a girl." This teacher considered this problem as a completely theoretical probability question and did not realise variation exists and the sample size will influence the variation. And, Teacher 28 CH didn't identify the key point of this question by saying "it is random and no absolute result".

#### *Criterion B (Interpretation of the intentions of official mathematics curriculum)*

When referred to official mathematics curriculum, Teacher 35 CH said, "This is the typical question representing thinking of probability and statistics. In the stage of primary school, statistics is more important. The main content of statistics is data processing, not to infer or guess with (theoretical) probability..."

Meanwhile, some teachers like Teacher 53 AU just listed several headings that are used in curriculum documents such as "measurement, chance and data" and some related ideas such as "calculating theoretical probabilities." Teacher 24 CH referred to "including mathematical thinking of abstraction, transformation, modeling and etc.," but could not identify any specific mathematical thinking implied in the questionnaire.

#### *Criterion C (Understanding of students' mathematical thinking)*

When teachers were required to comment on students' answers in Question 1(e) of Part A, Teacher 21 AU said "Students who choose C do not consider the variation but understand the basic principles of chance. Students who choose A, D, E and F, ... do not understand the basic principles of chance." This teacher gave reasonable

explanations for each response of students and was able to discriminate students' different thinking level on statistics.

But these "typical" answers of students were confusing for some teachers. For example, Teacher 4 CH, initially thought that "students' understanding is there are more red balls," but then pointed that "students will think all outcomes are possible, it's difficult to judge." This teacher did not understand the various misconceptions embedded in the "typical" answers.

#### *Criterion D (Design of teaching)*

When discussed on how to help students understand the critical mathematical thinking of Question 3, Part A, Teacher 54 AU articulated teaching plans: "There are many activities that can be carried out using counters, coins and dice to simulate certain events. In the case of babies being born male or female, tossing a coin 10 times and recording Heads as female and Tails as male could be done. If every student performs the 10 tries, I would have enough data to compare and expect a good range including possibly 80% female. I could then compare individual trials of 10 to collective trials by putting together 5 groups of 10 results and comparing the male and female numbers and hopefully show that the results tend more to 50:50 female: male," concluding "(one) would need to get across the idea that when an experiment is conducted many times over, certain patterns are likely to appear." This teacher correctly focused on the critical points and designed very elaborate simulation – coin tossing – which is more familiar to students, not just talking about general teaching strategies.

Teacher 19 CH indicated that the teaching focus was about "statistical knowledge," but offered no discussion of any specific statistical concepts, giving only very general teaching strategies like having "students conduct various kinds of experiments... they need practical manipulations to explore possibility." Likewise, Teacher 2 focused only on "understanding of fractions, percentages and decimals. I would introduce whole numbers and equivalence and converting decimals to percentages." This teacher referred only to some general strategies like "open ended questions including ratio of boys and girls."

#### **Quantitative analysis**

By assigning a score of 1 if one of the four indicators was evident in a teacher's response, and 0 if it was omitted from their response or answered inappropriately, it was possible to construct a score of 0 to 4 for each criterion, and hence a maximum score of 16 across the four criteria. We allowed for the possibility that teachers might provide convincing alternative indicators to the four indicators listed.

The two researchers operated independently to score teachers' responses; then a careful confirmative check took place in order to resolve any difference. A high degree of consistency was present in the initial grading by the two graders, where, in less than 30 cases of 0/1 grading, only minor differences occurred. Any resulting differences in grading the 82 responses across the four criteria were easily resolved by consensus.

#### *A summary for Chinese and Australian samples*

For the 41 Chinese teachers, the highest score was 14 and the lowest score was 3, with a median score of 8. For Australian teachers, the highest score was 15 and the lowest was

4, with a median score of 9. The respective mean scores were 8.34 (Chinese) and 9.27 (Australian) with standard deviations 2.70 and 2.63 respectively. Table 2 shows means and deviations that were calculated for each of the criteria and total score. Of the four criteria, Criterion D (Design of teaching) had the lowest mean (1.77) followed by Criterion B (Interpretation of the official mathematics curriculum) with the mean of 2.11, followed by Criterion C (Understanding of students' mathematical thinking) with the mean of 2.38. Criterion A (Knowledge of mathematics) had the highest mean at 2.56.

*Table 2 Means for each criterion and global means and deviations*

Sample	Criterion A	Criterion B	Criterion C	Criterion D	Total
Chinese(41)	2.42(0.67)	2.02(0.96)	2.27(0.71)	1.66(0.97)	8.37(2.70)
Australian(41)	2.72(0.86)	2.21(0.83)	2.49(0.76)	1.85(0.93)	9.26(2.63)
CH & AU(82)	2.56(0.82)	2.11(0.90)	2.38(0.76)	1.77(0.97)	8.82(2.79)

Australian teachers scored slightly higher on all four criteria than their Chinese counterparts, but there was no statistically significant difference. On Criterion A, Australian teachers were clearer on the understanding of critical concepts in statistics, especially in distinguishing variability from theoretical probability; on Criterion B, Chinese teachers paid more attention to methods to calculate possibility or chance that students need to learn, but Australian teachers were more focused on the development of how to deal with data in practical situations; on Criterion C, Australian teachers performed better in anticipating difficulties and misunderstandings that students might encounter; on Criterion D, Australian teachers were more likely to locate the key statistical idea in the hospital question in Part A, and could show in more practical ways how to develop related statistical thinking.

#### *Three classifications of teacher capacity*

Three sub-categories of our construct of teacher capacity were created, with the boundaries set on the basis of the qualitative analysis of teachers' responses as discussed earlier. These were High capacity (score 11-16), Medium capacity (score 6-10) and Low capacity (score 0-5). These classifications using the two samples are shown in Table 3.

*Table 3 Classifications of teacher capacity*

Capacity	Australian	Chinese
High	8(19.5%)	7(17.1%)
Medium	26(63.4%)	26(63.4%)
Low	7(17.1%)	8(19.5%)

There were one more High Capacity teachers in Australian sample than in Chinese sample (respectively 8 and 7); one less Australian teacher classified as Low Capacity than Chinese teachers (respectively 7 and 8). In both Chinese and Australian samples, Medium Capacity group was the biggest group, which included 26 teachers out of 41, that was more than 60%.

High Capacity teaching of statistical thinking was evident in nearly 20% of Chinese and Australian teachers' responses to the questionnaire. It was shown by a clear understanding of the critical thinking and concepts in statistics of the four questions of

Part A; relating the tasks to relevant curriculum documents; by high interpretative skills when applied to each of the six possible answers of students' work in Question 1(e), and; by an extensive range of ideas for designing and implementing a teaching program to support the development of students' statistical thinking. Medium Capacity was shown by approximately 60% of teachers who, while possessing knowledge and skills supportive of these directions, clearly need to increase their current levels of professional knowledge and skills. In both samples, Low Capacity was evident in a minority of teachers – nearly 20% – who appeared unable to express a clear articulation of the mathematical nature of the tasks, or what differentiated the six students' answers in Question 1(e). These teachers were unable to point with any confidence to how the tasks related to what is contained in official curriculum documents, and found it difficult to describe how they would plan a program of teaching to foster students' statistical ideas.

#### *Interrelations of teacher capacity*

Table 4 (calculated using SPSS 19.0 English version) shows the bivariate correlation between any two of the four criteria as well as that between each criterion and the total for the 82 Chinese and Australian samples.

*Table 4. Bivariate correlations between each criterion and the total and any two of four criteria*

		A	B	C	D	total
A	Pearson Correlation	1	.384**	.506**	.553**	.749**
	Sig. (2-tailed)		.000	.000	.000	.000
B	Pearson Correlation	.384**	1	.476**	.648**	.792**
	Sig. (2-tailed)	.000		.000	.000	.000
C	Pearson Correlation	.506**	.476**	1	.601**	.786**
	Sig. (2-tailed)	.000	.000		.000	.000
D	Pearson Correlation	.553**	.648**	.601**	1	.885**
	Sig. (2-tailed)	.000	.000	.000		.000
total	Pearson Correlation	.749**	.792**	.786**	.885**	1
	Sig. (2-tailed)	.000	.000	.000	.000	

\*\* . Correlation is significant at the 0.01 level (2-tailed).

First of all, all the four criteria have clear and strong contributions between .749 and .885 to the total score at the 0.01-level (2-tailed). And furthermore, all pairings of the four criteria have a significant correlation, supporting our theory that all four components of teacher capacity are interrelated. However, we cannot know whether the correlation between any two criteria was direct or was influenced by a third variable (Pallant 2001, p. 130). As a result, we statistically controlled one of the four criteria then carried out four partial correlation analyses between any two of the other three criteria. When Criterion A (Knowledge of mathematics) was controlled, the partial correlations between the other three criteria–B/C (.354), B/D (.567), and C/D (.447)–were all significant at 0.01-level. The result was the same when Criterion B (Interpretation of the intentions of the official mathematics curriculum) was controlled – A/C (.398), A/D

(.433) and C/D (.437). When Criterion C (Understanding of students' mathematical thinking) was controlled, the partial correlations between criteria A and D (.362) and between criteria B and D (.515) were significant at the 0.01-level. Only partial correlation between criteria A and B (.188) was the exception.

However, it showed very different results when Criterion D was considered as the statistically controlled variable. We can see from Table 5, the correlation between Criterion A and Criterion C was significant (.260) at 0.05-level, the rest of the pairs had no statistically significant correlations: A/B (.039), B/C (.142).

The results were slightly different from a previous research (Zhang & Stephens, 2013), but it was still obvious that after statistically controlling any one of the other three criteria, Criterion D still had significant and strongest correlation with the other two at 0.01-level. However, when Criterion D was statistically controlled, the relationship between any two of the other criteria became much weaker. A multi-step linear regression analysis confirmed the importance of Criterion D: on its own, Criterion D accounted for 88.5 % of the variance of our model. This also endorses our placing of Design of teaching at the center of the model shown in Figure 1, where it is informed by the other three criteria, which are only weakly related when dissociated from Design of teaching.

*Table 5. Partial correlations (criterion D as controlled variable)*

Control Variables			A	B	C
D	A	Correlation	1.000	.039	.260*
	B	Correlation	.039	1.000	.142
	C	Correlation	.260*	.142	1.000

\*Correlation is significant at 0.05 level

## Conclusions

Our construct of teacher capacity, presented here as teachers' professionally informed judgments and dispositions to act, connects to but differs from earlier research into pedagogical content knowledge by Shulman (1986; 1987) and Mathematical knowledge for teaching by Ball et al. (2008). Here teacher capacity was investigated in terms of knowledge of mathematics, interpretation of the intentions of official curriculum documents, understanding of students' thinking and design of teaching to foster the underlying mathematical ideas. Performance on each criterion was ascertained using a precise set of indicators that were related to the specific mathematical tasks, students' expected thinking in relation to those tasks, the relationship between the tasks and official curriculum documents, and teachers' ability to design explicit teaching sequences to support students' learning.

Design of teaching, informed by the other three criteria, appears to be the critical dimension for the implementation of curriculum reform; and the criterion that most clearly distinguishes between different levels of teachers' capacity to enact reform. Our construct of teacher capacity strongly reflects the view that effective implementation of

any curriculum reform depends on teachers' subtle interpretations of official curriculum documents and their professional dispositions to act on those ideas, which go well beyond general descriptions or statements of intent that are usually embodied in official curriculum advice.

Our construct of teacher capacity was built on earlier research into mathematical knowledge for teaching by MKT (Ball et al., 2008; Hill et al., 2008). However, to make the conclusions of this study compatible to their key idea of mathematical knowledge for teaching would appear to require a major reframing of their category, knowledge of content and teaching. In light of this study, that category appears to be too static and less suited to direct attention to design of teaching, which we have interpreted as understanding key aspects of national curriculum reform and knowing how to enact these aspects in practice. Our construct seems more suited to capture this key feature of curriculum reform.

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