Examining TIMSS items through the lens of Origami

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Introduction

With his invention of kindergarten in 1838, Friedrich Froebel used paper folding (origami) to introduce young children to informal explorations of mathematical concepts (Jarvis, 2009; Lister and Froebel, 2014). In the early years, his vision and methods were faithfully implemented, with their impact possibly realized in the work of noted artists and architects (Brosterman, 1997). Unfortunately, his intent was modified and misinterpreted with the growth and expansion of kindergartens around the world, resulting in the eventual abandonment of the use of origami in instruction in the early grades (Lister & Froebel, 2014). Today it is rare to see origami implemented systematically and systemically in kindergarten classes, or, for that matter, in mathematics classes across the grades in the U.S, although an Origametria program has been in place for several years in Israel under the direction of Miri Golan (Golan, in preparation.

In the 2011 administration of TIMSS (Trends in International Mathematics and Science Study) (Mullis et al., 2012b), the following grade 4 item (from the list of released items available at timss.org) was presented to students: *Tom ate half of a cake and Jane ate one quarter of the cake. How much of the cake did they eat altogether* (Item M041299)? Student scores ranged from a high of 84% to a low of 5% across countries; the average score was 23%. Might students introduced to origami in the early grades have performed at a higher level?

To consider this question, the authors examined a sample of released items to explore how paper folding might help improve student understanding of the concepts assessed. A few TIMSS items also were administered to students in two classes at grade 8 (one item) and grade 4 (two items), respectively. Students in one class at each grade then participated in an origami activity without discussion of the test items. Three weeks later the same items were re-administered to both the test class and control class at grades 4 and 8. This paper will examine a small number of sample items and countries' performance and discuss the results from this pilot study.

What is TIMSS?

Managed by the TIMSS & PIRLS International Study Center at Boston College, under contract to the International Association for the Evaluation of Educational Achievement (IEA), the Trends in International Mathematics and Science Study (TIMSS) is an international assessment in mathematics and science administered every four years to grade 4 and grade 8 students (www.timss.org)[1]; the assessments are based upon a framework developed by country representatives from the curricula of participating

[1] In 1995, 2008, and 2015, students from some countries also were assessed at the end of secondary school.

countries (Mullis et al., 2012a]. TIMSS 2011 included 74 education systems from six continents at both or one of the grades (see Fig. 1).

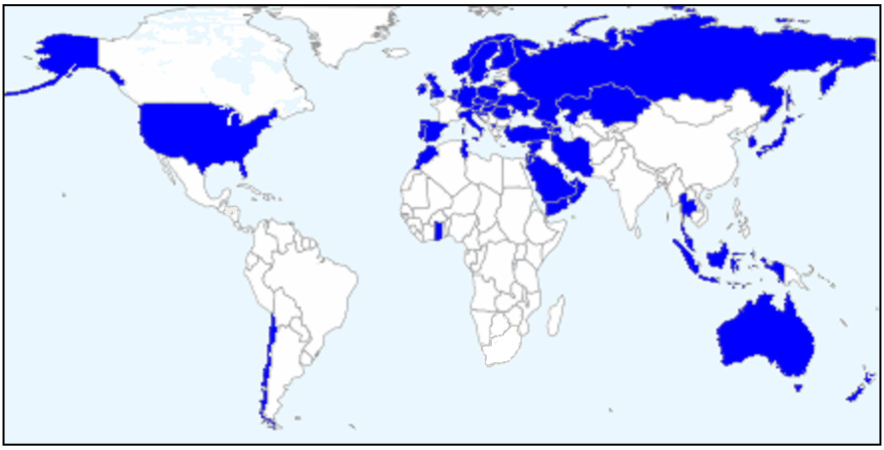


Figure 1. TIMSS 2011 Participating Education Systems

TIMSS data and results

With each administration of TIMSS dating back to 1995 and repeated every four years, released items are made available (timss.bc.edu/timss2011/downloads/T11_UG_G4_M_Items_Stats.zip) along with a detailed database about each participating country (Mullis et al., 2012c; Mullis et al., 2012d].

Presented in this paper are a few sample items for grades 4 and 8 and accompanying results from participating ASEAN member countries: Indonesia, Malaysia, Singapore, and Thailand, and other Asian economies: Australia, Hong Kong SAR, Japan, New Zealand, Russia, Republic of Korea, and Chinese Taipei. Also included are results from the U.S.

Table 1. Country Participation at Grades 4 and 8

Country	Grade 4	Grade 8
Australia	Yes	Yes
Hong Kong SAR	Yes	Yes
Indonesia	No	Yes
Japan	Yes	Yes
Korea, Rep. of	Yes	Yes
Malaysia	No	Yes
New Zealand	No	Yes
Russian Fed.	Yes	Yes
Singapore	Yes	Yes
C. Taipei	Yes	Yes
Thailand	Yes	Yes
U.S.A.	Yes	Yes

Item 1–Grade 4: Number; Fractions and Decimals; Knowing [Item M041299]
Tom ate ½ of a cake and Jane ate ¼ of the (same) cake. How much of the cake did they eat altogether?

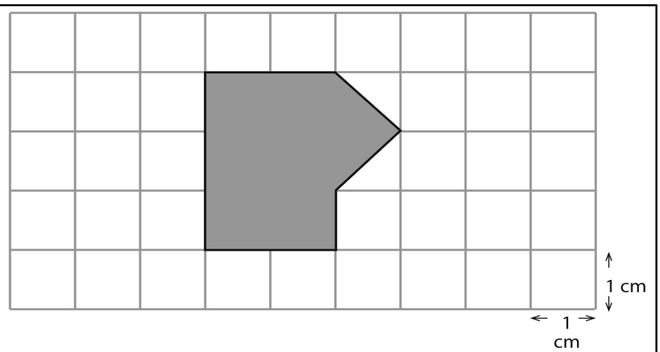
Table 2. Results for item 1

Country	**% Correct**	**Country**	**% Correct**
Singapore	84%	U.S.	35
C. Taipei	54	Japan	28
Hong Kong SAR	53	*International Av.*	23
Australia	37	Russian Fed.	14
S. Korea	36	Thailand	5

Seventeen percent of all students internationally selected the wrong answer of 2/6, which may have resulted from the addition of numerators and denominators, a common error made by students learning fractions. In fact, students and teachers identify fractions as concepts difficult to learn and challenging to teach. One may recall one's own experiences in learning fractions: finding equivalent fractions and common denominators, which clearly are most confusing to students. For the pilot study (see below), some students were presented with a square (rectangle also would work) sheet of paper and asked to identify the relationship of the original area to that produced when the paper was folded edge to edge. Students then were asked for the relationship when the edge of the paper was folded to the crease.

By simply looking at the paper, students were able to say the first fold produced two halves, and the second fold produced a section that was one-fourth of the original. They then were able to conclude that one half of the sheet and one fourth of the sheet equaled three quarters of the sheet.

Item 2–Grade 4: Geometric shapes and measures; 2– and 3–D shapes; Applying [Item M031297]



The squares in the grid above are 1 cm by 1 cm.
What is the shaded area in square centimeters?

Answer: ________________ square centimeters

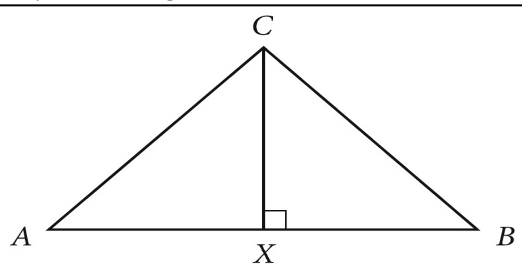
Table 3. Results for item 2

Country	% Correct	Country	% Correct
Japan	70%	Australia	43
Hong Kong SAR	67	Singapore	39
C. Taipei	63	U.S.	38
Korea, Rep.	48	Thailand	37
Russian Fed	44	*International Av.*	30

For item 2, students familiar with folding a square in half vertex-to-vertex might see easily that the triangle on the right side of the shape consists of two triangles that are each half of a square centimeter. An international average of 30% (Table 3) suggests that students have limited experience with visualization and concrete manipulations.

To reinforce students' familiarity with isosceles right triangles, the authors taught the students a model called kissing lips (instructions available online by searching for 'kissing lips').

Item 3–Grade 8: Geometry; Reasoning [Item M052362]



In this triangle:

$AC = BC$

AB is twice as long as CX.

What is the size of angle B?

Answer: ________________ °

To be able to answer correctly the question in item 3, students must know: 1) the symbol for a right angle; 2) a right angle is 90^{o}; 3) the sum of the internal angles of a triangle is 180^{o}; 4) definition of an isosceles triangle; 5) ∠CXB is a right isosceles triangle. Through origami, students can learn all these properties easily and painlessly while folding different models that begin with folding a square in half vertex-to-vertex.

Table 4. Results for item 3

Country	% Correct	Country	% Correct
Korea, Rep.	89%	*International Av.*	41
Japan	85	New Zealand	40
Singapore	83	U.S.	39
Hong Kong SAR	72	Thailand	39
C. Taipei	73	Malaysia	28
Russian Fed.	52	Indonesia	18
Australia	48		

Item 4–Grade 8: Geometry; Applying [Item M042270]

The length of side of each of the small squares represents 1 cm. Draw an isosceles triangle with a base of 4 cm and a height of 5 cm.

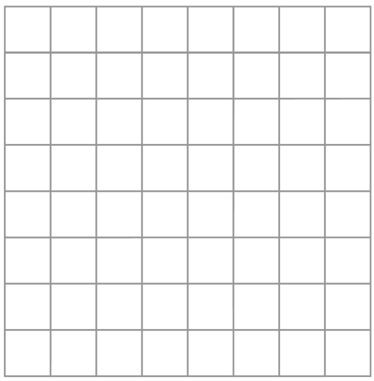
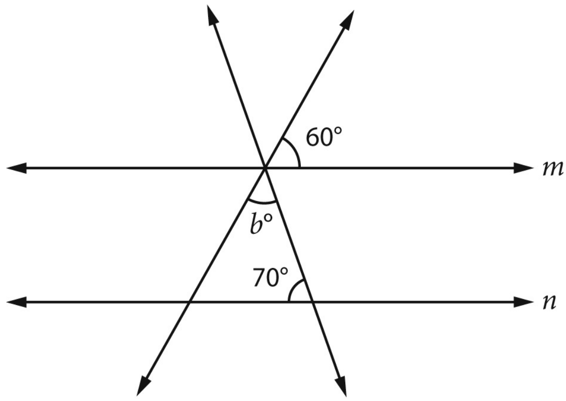


Table 5. Results for item 4

Country	% Correct	Country	% Correct
Japan	85%	New Zealand	46
Korea, Rep.	84	Australia	41
Hong Kong SAR	82	Malaysia	39
C. Taipei	82	Indonesia	39
Russian Fed.	75	Thailand	30
Singapore	72	U.S.	27
International Av.	48		

With an international average of 48% of students providing a correct response (Table 5), one may conclude that over half of students may not have had experience with constructions, and/or they don't know what an isosceles triangle s, a concept with which students familiar with paper folding may be quite familiar.

Item 5–Grade 8: Geometry; Reasoning [Item M052408]



Lines *m* and *n* are parallel.

What is the value of *b*?

Table 6. Results for item 5

Country	% Correct	Country	% Correct
Japan	86%	Malaysia	32
Korea, Rep.	85	Thailand	29
Singapore	80	Australia	28
Hong Kong SAR	75	U.S.	24
C. Taipei	49	New Zealand	25
Russian Fed.	48	Indonesia	20
International Av.	33		

Pilot study and results

The wide range in student performance across the above five items – from a high score of 89% on item 3 to a low score of 5% on item 1 – prompted the question: Might students introduced to origami in mathematics instruction have performed at a higher level? To examine this question, TIMSS items 1, 2, and 3 were administered at one school to students in two classes at grade 4 (items 1 and 2) and grade 8 (item 3), respectively. Students in one class at each grade then participated in paper folding activities (described above) during one period without any reference made to the test items. Three weeks later the items were re-administered to both the test class and control class, which had not been exposed to paper folding. Results are tabulated in Table 7.

At grade 8, both classes performed at or above the international average of 41%, on the pre-assessment, whereas the U.S. score was 39% (see Table 4). The control class exhibited no difference in performance on the post-assessment, whereas an additional three students in the test class registered a correct response on the post-assessment, raising the performance to 67%.

Table 7. Results from pre- and post-assessment

Grade	Item 1-pre	Item 1-post	Item 2-pre	Item 2-post	Item 3-pre	Item 3-post
4E-test	9/19*	12/19	4/19	9/19	---	---
4B-control	5/12	6/12	1/12	2/12	---	---
8B-test	---	---	---	---	7/15	10/15
8A-control	---	---	---	---	10/19	10/19

Item 1: Adding fractions; Item 2: Finding area; Item 3: Finding angle in a triangle.
*# correct/total number of students

At grade 4, the students in the test class performed better than the control class on the pre-assessment, with 50% and 21% providing correct responses on the pre-assessment for items 1 and 2, respectively. The control class performance was 42% and 8%, respectively. The U.S. averages were 35% and 38%, respectively. The students at this school performed above the international average on the fractions items but did less well in finding the area of the shape.

The post-assessment data suggest a possible effect of the paper folding activity. For the test class, three more students were able to respond correctly in the item 1 post-assessment; in the control class, the correct response increased by one student. A more pronounced difference was observed for item 2, where an additional five students were able to provide a correct response in the post-assessment. In the control class, an additional student provided a correct response.

Conclusion

The original intent of this paper initially was limited to examining some publicly released TIMSS items and discussing ways in which paper folding might help students develop a better understanding of the mathematics concepts presented in the items. A small pilot study then was developed to actually measure student performance on the items before and after engagement with paper folding activities.

Earlier studies have attempted to examine the effects of paper folding on students' spatial and visual abilities, but without utilizing actual test items (Boakes, 2011; Çakmak, 2009). In this paper, the authors conducted a small pilot study to gauge whether a difference could be observed in students' mathematical knowledge. No definitive conclusions can be drawn from a pilot study with such a small number of students, but data for both grades 4 and 8 suggest a possible effect of paper folding on student learning.

The paper folding activity took place only for an hour in this pilot study. What differences might be observed on standardized tests if students were exposed to paper folding as part of their mathematics experience for a year or throughout their schooling? Paper folding not only can hone students' understanding of mathematics concepts, but it can help improve their manual dexterity, visualization, and attention to precision.

Item 1 illustrates that origami can be useful also for areas of mathematics beyond geometry. In *Project Origami* Hull provides many examples of the application of paper folding to different areas of mathematics (Hull, 2012).

From examination of the TIMSS items and discussion of ways in which paper folding might help improve the performance particularly of low scoring students, one can speculate that perhaps even informal paper folding outside the classroom may help improve student learning if those providing instruction in paper folding were to make an attempt to mathematize the process of paper folding. Currently many origami instructors use non-mathematical terminology in teaching, such as corner, diaper fold, book fold, and cupboard fold.

The corner of a polygon is a vertex, which on a square is a right angle. A diaper fold is folding a square in half vertex-to-vertex to form a right isosceles triangle. In the process, one bisects the opposite right angles. A book fold is folding a square in half edge-to-edge. A cupboard fold consists of folding a square into equal fourths. Many instructors name a four equal-sided polygon sitting on an edge a square. However, when it is turned with a vertex pointing down, they call it a diamond. Regardless of orientation, a square is a square.

In conclusion, those in the origami community can make a significant impact in improving student appreciation of and learning of mathematics.

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