Seeking high quality collaboration and reflection: A Lesson Study on primary school mathematics

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1. Introduction

The Japanese lesson study model and its typical process are well known internationally (e.g. Stigler & Hiebert, 1999; Lewis, 2002; NASEM, 2011). The success of lesson study can be found in two primary aspects: improvements in teacher practice and the promotion of collaboration among teachers. Lesson study provides Japanese teachers with opportunities to make sense of educational ideas within their practice, to change their perspectives about teaching and learning, and to learn to see their practice from students' perspectives (Takahashi, 2010). There is, however, a few in detail information about recent Japanese mathematics lesson study (Isoda, et al., 2007; Shimizu, 2010; Takahashi, 2010, 2014).

Lesson study takes different forms including school-based lesson study, district-wide lesson study, and cross-district lesson study. Koyama (2012) focused on a school-based and district-wide lesson study on primary school mathematics for three years at a local public primary school in Japan in identifying an example of 'good practice' of teaching and learning of mathematics. In this paper, as a school-based and cross-district lesson study, we will look at the whole process of the lesson study on primary school mathematics at the Hiroshima University Attached Primary School where the author as a colleague and mathematics educator has been collaboratively working with mathematics teachers for several decades since the 1990s. The purpose of this paper is to rethink a lesson study on school mathematics for seeking high quality collaboration and reflection as an authentic area in the science of mathematics education.

2. Background information

We first note some fundamental information needed to understand Lesson Study in Japan (Koyama, 2004, 2008a, 2008b, 2010; Ministry of Education, 2008a, 2008b).

- The school education system is composed of six-year primary school, three-year lower secondary school, and three-year upper secondary school. The first two levels for nine years, age 6 to 15 years old, are compulsory education for all children.
- The school year begins on 1st April and ends on 31st March of the following year.
- The national curriculum standard is prescribed in the 'Course of Study' determined and issued by the Ministry of Education, Culture, Sports, Science and Technology (*Monbukagakusho*). The 'Course of Study' provides the basic framework for the mathematics curriculum; the required time spent on mathematics, the overall aim of mathematics, and the objectives and contents of mathematics teaching and learning at each grade in school.
- The school textbooks must be approved by the Ministry of Education according to the 'Course of Study'.
- The teaching practice in pre-service teacher training is mainly undertaken by university-attached schools.
- Public school teachers are local prefectural or municipal public officials and

are appointed by the respective local prefectural or municipal boards of education in which the schools are located.
- Primary school teachers teach almost all of school subjects at their own grade in the classroom. They are not necessary specialists in mathematics education.

The Hiroshima University Attached Primary School has been attached to Hiroshima University for more than 100 years to accomplish its special mission of taking a part of teaching practices for pre-service teacher training in Hiroshima University, doing new developmental research for the next generation education, and doing lesson study and opening classes to teachers from all districts in Japan as well as teaching own students of the school. This national school is very unique because it has mathematics teachers who teach only mathematics at some different grades while primary school teachers usually teach almost all of school subjects at their own grade. More than 100 years the school has been issuing its own monthly journal *School Education Journal* which runs a pair of articles on lesson study on primary school subject by an attached school teacher who conducts a research lesson and by a Hiroshima University educator who is a specialist in the school subject education. In addition, we have been organizing a collaborative study team for the periodic seminar on primary school mathematics with not only mathematics teachers and mathematics educators but also graduated school students of mathematics education PhD and Master courses in Hiroshima University.

3. A typical model of good practice

A typical model of good practice recognized by many Japanese educators and teachers is the 'problem solving lesson' in the mathematics classroom (Becker & Shimada, 1997; Stigler & Hiebert, 1999; Burghes & Robinson, 2009). The 'problem solving lesson' is structured and progressed through four distinct phases; presentation of the problem, developing a solution, progressing through discussion, and summarizing the lesson (Burghes & Robinson, 2009, p. 56).

Why do they recognize the 'problem solving lesson' as a good model for teaching and learning mathematics? The essence of doing mathematics is the process of solving a problem mathematically rather than its product. If they are acquired in the process of solving problems mathematically, then we believe the mathematical knowledge, skills, and ways of mathematical thinking are applicable in a new or unfamiliar situation for us. Therefore in the mathematics classroom, mathematics lessons as an interaction of teacher's teaching and students' learning activities should be structured and progress through the process of solving problems mathematically. In other words, the 'problem-solving lesson' is a student-centered approach that could encourage students to construct mathematics collaboratively by using their acquired mathematical knowledge, skills, and ways of mathematical thinking in the classroom.

As Sawada (1997) points out, the advantage of using the 'problem-solving lesson' model for teaching and learning primary mathematics in the classroom are:
- Students participate more actively in the lesson and express their different ideas or solutions more frequently.
- Students have more opportunities to make comprehensive use of their knowledge, skills, and ways of thinking.
- Even low achieving students can respond to the problem in some significant

ways of their own.

- Students are intrinsically motivated to give their justifications or proofs.
- Students have rich experiences in the pleasure of mathematical activities and receive the approval from peer students in the classroom.

4. Whole process of lesson study in the 2nd grade

The lessons at the Hiroshima University Attached Primary School are structured using the 'problem-solving lesson' model in teaching and learning of primary school mathematics emphasizing students' continuous awareness of learning mathematics. We show one example of lesson study conducted by the collaborative study team at the Hiroshima University Attached Primary School from November 2012 to February 2013 in the third semester of academic year 2012. The lesson adopts the 'problem-solving lesson' model in teaching and learning of 'Triangle' in the teaching unit 'Triangle and Quadrangle' for primary school 2nd graders. As one of open classes in the annual Open School Conference, the research lesson was conducted with his 40 classroom students by Mr. Kazushige Maeda, an experienced mathematics teacher with 18 years of teaching experience as a primary teacher with a Master's degree, and observed by about 200 teachers from all districts in Japan on 9th February 2013. Figure 1 shows the whole process of this lesson study beginning from the collaborative study on teaching materials for the research lesson, planning the research lesson, conducting the research lesson, post-lesson discussion, to writing a pair of journal articles of reflections on the lesson study.

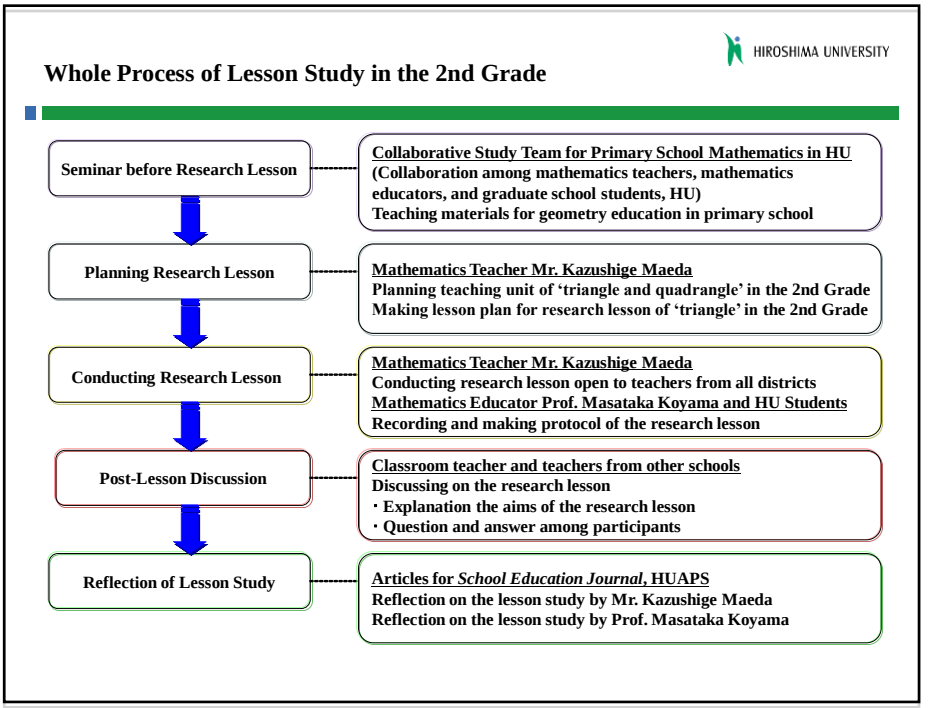


Figure 1. Whole process of lesson study in the 2nd grade at HUAPS

In the following sections, we will look at the lesson study in three categories; the collaborative study on teaching materials for research lesson, the lesson plan of research lesson, and the analysis and reflection of lesson study. It could help us rethink a lesson study on school mathematics for seeking high quality collaboration and reflection as an authentic area in the science of mathematics education.

5. Collaborative study on teaching materials for research lesson

5.1 Roles of collaborative team members

In November 2012, our study team called *Primary School Mathematics Seminar* began the collaborative study on teaching materials for teaching and learning of 'Triangle' in the teaching unit 'Triangle and Quadrangle' for primary school 2nd graders. We had four 90-minute sessions on 22nd November 2012, 4th December 2012, 20th December 2012, and 10th January 2013 before the research lesson. The team consisted of the mathematics teacher, three graduate students of mathematics education PhD and Master courses, and two mathematics educators in Hiroshima University. The team did collaborative studies on teaching materials in order to help the mathematics teacher plan the teaching unit 'Triangle and Quadrangle', and develop the research lesson plan of 'Triangle' for his 2nd graders.

In the team, the role of the mathematics teacher is first to decide on the mathematics topic for lesson study, to plan a series of lessons on the topic, and to make a draft lesson plan of one-unit-hour (i.e. 45 minutes) on certain content for the research lesson. Then after each session, the teacher modifies his draft lesson plan in taking the group discussion into consideration for the next session. On the other hand, the role of the graduate school students is first, after receiving the draft lesson plan from the mathematics teacher, to investigate the related research findings in mathematics education both from a theoretical and a practical perspective, and then in each session with the mathematics teacher, to share the findings with group members in order to improve the lesson plan. The mathematics educator does not control their discussion but coordinates the sessions and makes clear the main objectives of the research lesson and how they can be realized in the lesson with 2nd graders.

5.2 Records of the four sessions

In the first session on 22nd November 2012, the mathematics teacher proposed the students' activity to identify whether a figure is triangle or not so that his students capture triangle in terms of its components. He also shared with team members his expectations to be realized during this activity. The main issue of the team discussion was determining a criterion for the students to use in judging whether a figure is triangle or not. Finally the team members recognized the necessity to investigate the mathematical background related to the definition of triangle and some research findings about the role of counterexamples (i.e. the role of negation) in students' concept formation of triangle.

In the second session on 4th December 2012, through the team discussion, it was confirmed that the activity proposed by the mathematics teacher in the last session could lead to helping the students learn a fundamental mathematical activity such as going back to the definition if needed. Through the activity the students may be promoted to think about the reason why the triangle is not defined as a figure connected by three different points. In the latter part of the session, the team members discussed an

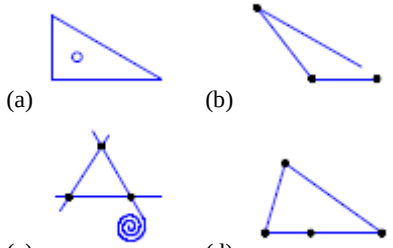
alternative activity such as asking the students "Can you see a triangle in the given figure?" being more appropriate for the students than asking "Is the given figure a triangle or not?" Moreover, the team members recognized the necessity to design the development of the research lesson in more detail.

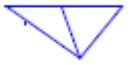
In the third session on 20th December 2012, it was confirmed that the research lesson could be supported theoretically on the basis of the philosophy of mathematics education and recognition theory of geometric figures. As a result, the team members agreed that the research lesson could contribute to deepen the students' recognition and understanding of triangle. On the other hand, the team members discussed how the students would interpret "Let's classify the given figures into two groups." In the latter part of session, the mathematics teacher proposed eight different figures that can be possibly used for the research lesson. The team discussed how to select and order some of them in the research lesson to make it most effective for the students in learning triangle.

In the fourth session on 10th January 2013, the main issue of the team discussion was how to deepen the students' understanding of the definition of triangle already learnt as the geometric figure surrounded by three straight lines in the previous lesson in the 2nd grade. The aim was to determine how to organize both the teacher's activity of 'shaking' students' recognition and understanding of triangle and the students' activity of rethinking the definition of triangle. In order to promote these activities, the team members shared and discussed the possible ways of showing the prepared figures to the students, and the expected advantage and disadvantage of giving a counterexample of a triangle to the students in the research lesson.

6. Lesson plan of the research lesson

After the four sessions, Mr. Kazushige Maeda, the mathematics teacher of the research lesson, planned the series of 15 lessons in the teaching unit 'Triangle and Quadrangle' for 2nd graders of primary school. Then he located the research lesson at the 5th of 15 lessons, and developed the lesson plan as shown in Figure 2.

Learning activities	Learning contents	Teaching remarks
■ <u>Posing the task</u> 1. Students classify the given figures into two groups; the one which has a triangle and the other which does not have a triangle.	<u>Problem</u> Classify the following figures into two groups.  (a) (b) (c) (d)	○ Teacher prepares the figures which highlight vertices and sides. ○ Teacher includes some figures which could be seen in different ways. ○ Teacher shows the figures one by one, and writes down the

	  (e) (f) ・To understand the activity of classifying given figures into two groups; the one in which we can see a triangle or the other in which we cannot see a triangle. ・To see the given figure in terms of its components (i.e. vertices and sides). ・To remember the definition of triangle as the geometric figure surrounded by three straight lines.	students' viewpoint of the figure on the blackboard. ○ Teacher prepares the sheet of paper on which each figure is drawn to move it in the activity. ○ Teacher writes down keywords related to the definition of triangle and representing the students' viewpoint of the figure on the blackboard.
2. Students have the task for today's lesson. ■ Exploring the task	Think about the reason why you classified the figure in such a way.	
3. Students think about the reason of their classification using components of the given figure.	・To see the components of the figure, and to make a judgment from the definition of triangle.	○ Teacher will make a figure with sides and vertices if needed according to the students' idea.
	・To notice that the judgment can be changed depending on the viewpoint. ・To think about the reason why we do not say that the figure connected three points is a triangle. Because when there are three points on a line, the figure connected them does make a triangle. Because the figure is not surrounded even though three points are connected.	○ Teacher asks students "Why do not you say that the figure connected three points is a triangle?" in order to deepen students' understanding the definition of triangle.

Expected reason why we can see a triangle	Expected reason why we cannot see a triangle
Fig.(a): Because there are three vertices. Because there are three sides. When ignoring the inside small hole.	Fig.(b): Because being not surrounded by sides. Because a point and a side are disconnected.
Fig.(c): Because there are three vertices. Because there are three sides. When ignoring the part of spiral.	Fig.(c): Because a side is not straight. Because there is the part of spiral shape.
Fig.(d): Because being surrounded by three sides.	Fig.(d): Because there are four points. Because there are four sides.

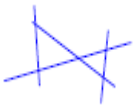
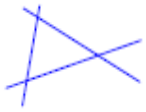
■ Clearing the task 4. Students summarize the reason of their classification, and tackle the application problems of classifying Fig.(g) and Fig.(h) .	・To confirm that we look at vertices and sides in the figure in our judgment. ・To reconfirm the definition of triangle.   (g) (h) ・To say that we can see a triangle in Fig.(g) and Fig.(h) by the definition of triangle.	○ Teacher will reinforce students' understanding of triangle through making a connection between the definition of triangle and keywords used for their judgment and the given figure.

Figure 2. Lesson plan for the research lesson 'triangle' in the 2nd grade at HUAPS

We must remember the fact that before this research lesson the 2nd graders already learned the definition of triangle in the teaching unit 'Triangle and Quadrangle' such that the geometric figure surrounded by three straight lines is called triangle. In the final version of the lesson plan (Figure 1), Mr. Maeda, mathematics teacher of the research lesson, intends to deepen his students' recognition and understanding of triangle. There are three notable educational features of this lesson plan. The first feature is using different figures including delicate figures of Fig. (c) and Fig. (d) as teaching materials. The second is the way of showing these figures to the students, one by one beginning from Fig. (a), Fig. (b), Fig. (c), and so on in order to stimulate the students to remember the definition of triangle and to use the definition in explaining the reason for their judgment. The third is incorporating the key question "Why do you not say that the figure connected by three points is a triangle?" in order to 'shake' the students' recognition of triangle and to deepen their understanding the definition of triangle. It is sure that these three features are crystallized as a result of the collaborative team study on teaching materials in four sessions held at the Hiroshima University Attached Primary School from November 2012 to January 2013 before the research lesson in February 2013.

7. Analysis and reflection of lesson study

The research lesson was conducted by Mr. Maeda on 9th February 2013 as one of open classes in the annual Open School Conference, where about 200 teachers from all districts participated to observe the lesson. The lesson and the post-lesson discussion were video-recorded and photographed by the graduate school students of the collaborative team, and the author also observed the lesson and took a field notes during the lesson observation. After the research lesson, the detailed protocol of the lesson was made and used by both Mr. Maeda and the author in writing a pair of articles of reflection on the lesson study submitted to the monthly journal *School Education Journal* issued by the Hiroshima University Attached Primary School. The articles were published about one year later in January 2014 (Maeda, 2014; Koyama, 2014). The publication of a pair of articles of reflection on the lesson study is very unique and important in the sense that it can contribute to promote the lesson study on school

mathematics not only as a means of the continuous professional development of teachers but also as an authentic area in the science of mathematics education.

In this section, the lesson study will be analyzed from the observer's perspective to obtain some implications for improving the teaching practice of primary school mathematics. In the analysis and reflection, the observer of the lesson (the author of this paper) uses the data of the research lesson such as the lesson plan, the detailed protocol of the lesson, and field notes as well as the article of reflection on lesson study by the mathematics teacher (Mr. Maeda) who conducted the research lesson.

7.1 The teacher's perspective on geometry education in primary school mathematics

The lesson study analyzed in this paper is one new proposal for the geometry education in the 2nd grade of primary school in Japan. It is based on the teacher's perspective on education as follows:

> *The aim of teaching and learning geometry in primary school mathematics is to foster students' ability of mathematical thinking. Therefore in the lesson study the teacher aims at fostering his students' ability of logical thinking especially by the activity for stimulating the students to think about the definition and properties of geometric figure. In other words, it aims that the students become able to think logically and represent their idea* (Maeda, 2014, p. 45).

This perspective of the teacher is analyzed by using two concepts of the immutable and fashionable in the history of geometry education in primary school mathematics in Japan. As a result, two features of the perspective are identified. The first feature relates to the immutable character of geometry education. In the Course of Study by the Ministry of Education in Japan, two objectives of geometry education have been immutable in school mathematics: One is developing geometric concepts and understanding properties of geometric figures, and the other is fostering students' ability of logical thinking. Usually in the primary school mathematics the former is more emphasized than the latter while the latter is more emphasized than the former in the secondary school mathematics. In the lesson study, however, the teacher claims that the latter should be more emphasized than the first and could be realized even at lower grades in the primary school mathematics. The second feature reflects the current fashion of school education in Japan. The Course of Study revised in 2008 emphasizes the necessity of fostering students' ability to 'think, judge, and represent' in all school subjects including mathematics in order to overcome Japanese students' weakness revealed by the national and international surveys on students' attainment. In the lesson study, the teacher aims at fostering his students' ability to think and represent their idea mathematically in accordance with the current school curriculum policy and emphasis. It might be said that the first feature is the main proposal by the teacher with his 2nd graders in the lesson study on primary school mathematics.

7.2 Three proposals of the lesson study

In taking the teacher's proposal into consideration, the lesson study is reflected by the observer. In the research lesson, the teacher intended to deepen his students' recognition and understanding of triangle. As a result of the collaborative team study on the teaching materials, the following three points were proposed to be examined and evaluated through the research lesson: (1) the use of eight different figures as teaching

materials and the way of showing these figures to the students one by one beginning from Fig. (a), Fig. (b), Fig. (c), and so on; (2) activities of classifying and identifying the given figures from different viewpoints of the figure, and; (3) incorporating the key question "Why do you not say that the figure connected by three points is a triangle?" in order to 'shake' the students' recognition of triangle.

The research lesson was structured by the use of eight different figures (see Figure 2) as teaching materials and the way of showing these figures to the students one by one beginning from Fig. (a), Fig. (b), Fig. (c), and so on as follows: The definition of triangle (the students already learned), showing Fig. (a) and Fig. (b) for confirming the definition of triangle, showing Fig. (c) for being aware of viewpoints of figures, showing Fig. (d) for being aware of counterexamples, showing Fig. (e) for paying attention to vertices, showing Fig. (f) for paying attention to sides, and showing Fig. (g) and Fig. (h) for reconfirming the definition of triangle leading to deepen the students' understanding of the definition of triangle (Figure 3). In the teaching and learning of triangle, Fig. (c) and Fig. (e) are not usually used while Fig. (b) and Fig. (e) are usually used by primary school teachers in Japan. The use of Fig. (c) and Fig. (d) reflects the teacher's emphasis on the viewpoints of figure and counterexamples in the research lesson. It must be a unique proposal of the research lesson in terms of structuring a mathematics lesson.

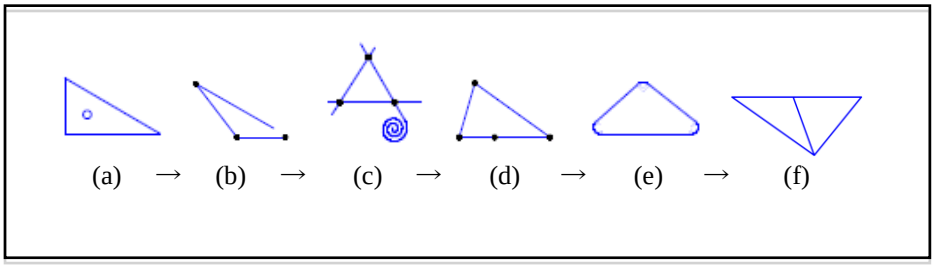


Figure 3. Structure of the research lesson in the 2nd grade at HUAPS

In the light of structuring a mathematics lesson, another prominent proposal is to organize social interactions in the research lesson. The teacher expected that if Fig. (c) was shown then the students had opposite opinions about whether the figure is a triangle or not. Moreover the teacher intended to 'shake' the students' recognition of triangle by the key question "Why do not you say that the figure connected three points is a triangle?" as a counterexample. These two occasions are important opportunities for the students to change their viewpoints of figure and to reconfirm their understanding of the definition of triangle through social interactions such as pair talking and whole classroom discussion in the lesson.

7.3 Importance of viewpoints, counterexamples, and roles of mathematics teacher

The research lesson conducted by the teacher was analyzed by using the protocol of the lesson and the picture of the blackboard. As a result of the analysis, the author found the importance of viewpoints of figure, counterexamples, and the different roles of the teacher in two notable scenes of the lesson.

In the first notable scene where Fig. (c) was shown by the teacher, the students had opposite judgments about whether the figure is a triangle or not. After one-minute pair talking, the students exchanged actively their views about the figure in the whole classroom discussion. Some students presented their opinions in their own words. For example, a student said "Those who judged Fig. (c) as not a triangle see the figure as a whole including a part of spiral, on the other hand those who said Fig. (c) has a triangle see inside a part of the figure surrounded by three straight lines." During the classroom discussion, the teacher did not comment about the students' opinions or presentations but wrote down only some keywords related to their view about the figure on the blackboard such as "If we change our viewpoints," "It depends on the viewpoint of figure," and "If we see the figure like this" (Figure 4). In this scene the supportive role of the teacher functioned effectively for the students to exchange their judgments on the definition of a triangle and be aware of different views about the figure.

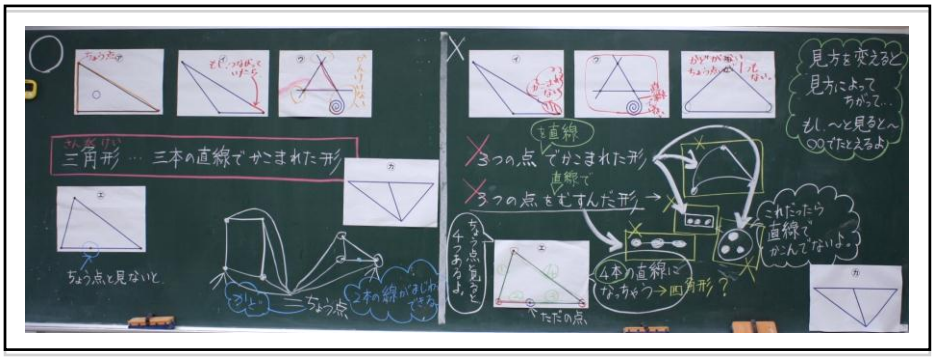


Figure 4. Blackboard of the research lesson in the 2nd grade at HUAPS

The second notable scene of the lesson is related to the discussion between the teacher and the students. The scene began with the teacher's asking the students "Why do you not say that the figure connected by three points is a triangle?" Immediately many students objected to what the teacher said. A student explained the reason why the figure connected by three points is not a triangle by drawing the counterexample A (Figure 5) like that "If the figure is surrounded by three points does not make a triangle. Three points must be connected by straight lines". Then the teacher countered, "I see. Do you agree that the figure connected by three points with a straight line is a triangle?" Another student did not agree with the teacher, and refuted by drawing the figure of counterexample B (Figure 5) with three points on the same straight line like that "I do not agree. If three points connected by the straight line was a triangle, then this (counterexample B) should be a triangle. Therefore a triangle must be the figure surrounding three points by straight lines." The student insisted the importance of being surrounded by straight lines. At this moment, the students in the classroom seemed to be satisfied with their peer's refutation. However, the teacher with a smile drew a figure (counterexample C) on the blackboard. In this scene, it can be said that the active intervention of the teacher functioned effectively for the students to deepen their understanding the definition of triangle through the discussion with counterexamples.

8. Conclusion

In this paper, we looked at the whole process of the lesson study on primary school mathematics at the Hiroshima University Attached Primary School. The lesson study was analyzed and reflected from the observer's perspective to obtain some implications for improving the teaching practice of primary school mathematics. As a result of the analysis and reflection of the lesson study, the following are implications. The first is the importance of incorporating viewpoints of figure and counterexamples with the appropriate teaching materials in a mathematics lesson for 2nd graders. The second is the fact that there are two different 'supportive' and 'active' intervention roles of a teacher and these roles should be organized appropriately in a mathematics lesson to promote students' social interactions and to realize the objectives of a mathematics lesson. The last but not least is the importance of high quality collaboration in a study team for lesson study on school mathematics in order to rethink the philosophy and perspectives of teaching and learning mathematics and make clear objectives of learning a particular mathematical content, to investigate useful teaching materials and problems (tasks) to be posed in a mathematics lesson, to think about different teaching methods appropriate for both students and a topic to be taught in the lesson, and to think about and choose how to word key questions in the lesson.

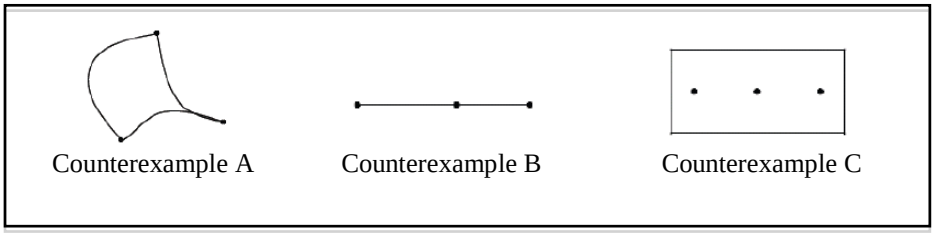


Figure 5. Counterexamples presented in research lesson in the 2nd grade at HUAPS

The lesson study on school mathematics has been recognized as an important cultural and collaborative means for the continuous professional development of teachers as a life-long process. At the same time, the author insists the lesson study is an authentic area in the science of mathematics education. In order to realize it we need to seek further high quality collaboration and reflection nationally and internationally in the lesson study on school mathematics.

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Endnotes

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