Supporting Teachers in Structuring Mathematics Lessons Involving Challenging Tasks

Peter Sullivan
Monash University, Australia

Abstract

- This presentation reports on aspects of a project that is
  - exploring the characteristics of tasks that are appropriately challenging for school students, and
  - ways of supporting teachers in converting such tasks into classroom lessons.

Proposition Set 1

- Students benefit from working on tasks that they do not already know how to do
- Students are more likely to connect ideas if they compare and contrast related ideas and build networks of concepts for themselves
- These connections are the key to remembering and transferring knowledge
- There are risks if we build connections too slowly


CAUTION
WATER ON ROAD
DURING
RAIN

Where does the idea of “challenge” come from?

- Guidelines for school and system improvement (see, e.g., City, Elmore, Fiarman, & Teitel, 2009)
- The motivation literature (Middleton, 1995; 1999).

The argument for challenge

- Learning will be more robust if students connect ideas together for themselves, and determine their own strategies for solving problems, rather than following instructions they have been given.
- Both connecting ideas together and formulating their own strategies is more complex than other approaches and is therefore more challenging.

This connects to “mindsets”

- Dweck (2000) categorized students’ approaches in terms of whether they hold either *growth* mindset *or fixed* mindset

Students with *growth* mindset:

- Believe they can get smarter by trying hard
- Such students
  - tend to have a resilient response to failure;
  - remain focused on mastering skills and knowledge even when challenged;
  - do not see failure as an indictment on themselves; and
  - believe that effort leads to success.

Students with *fixed* mindset:

- Believe they are as smart as they will even get
- Such students
  - seek success but mainly on tasks with which they are familiar;
  - avoid or give up quickly on challenging tasks;
  - derive their perception of ability from their capacity to attract recognition.


A

Teachers can change mindsets

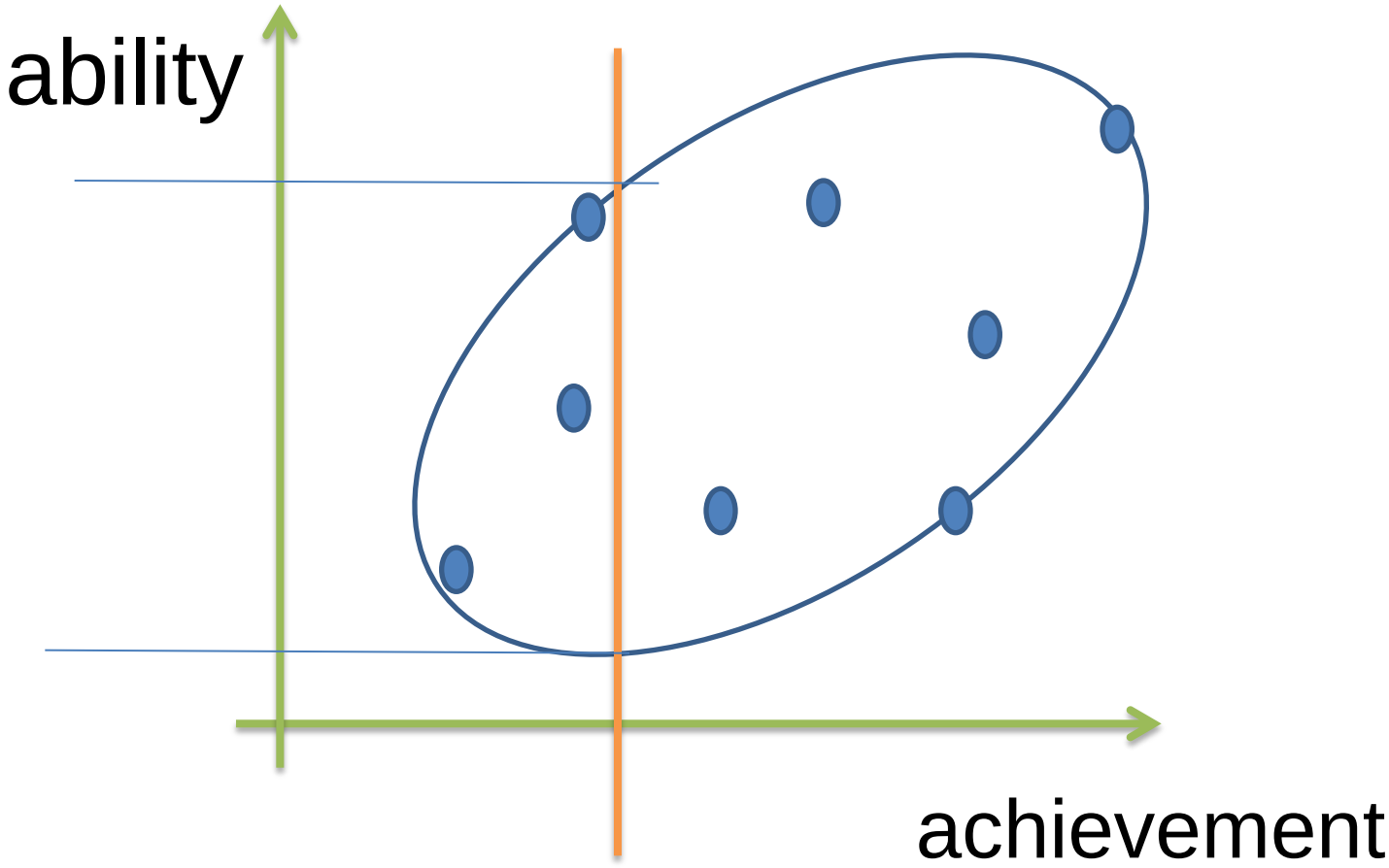
- the things they affirm (effort, persistence, co-operation, learning from others, flexible thinking)
- the way they affirm
    - You did not give up even though you were stuck
    - You tried something different
    - You tried to find more than one answer
- the types of tasks posed

Proposition Set 2

- Posing challenging tasks requires a different lesson structure
- This is relevant whether or not the students are grouped by their achievement
- And is applicable with crowded (and even badly behaved) classrooms


ENTRANCE
ONLY
DO NOT
ENTER


SLOW DOWN
OR
DIE

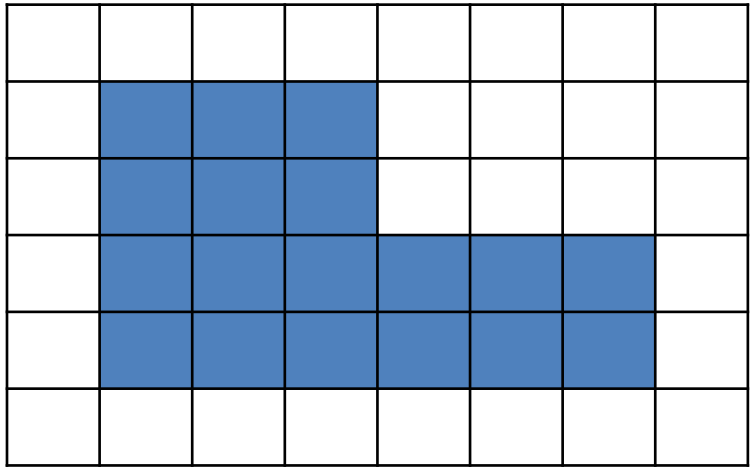

ability
achievement

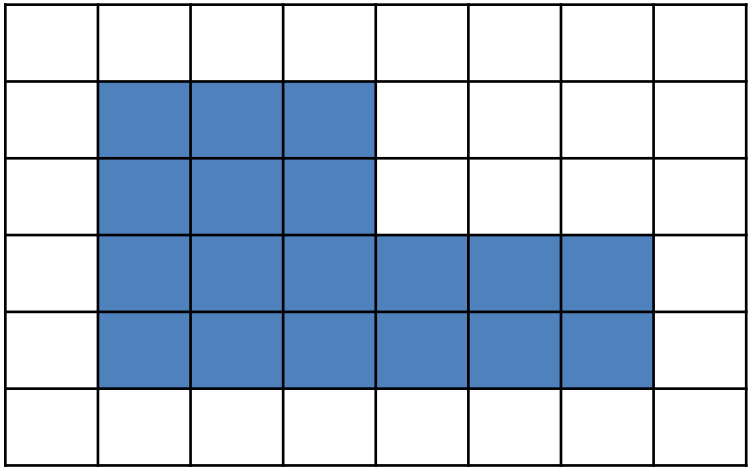
The conventional mathematics lesson "chalk and talk"

- Review homework
- Explain the concept and model the techniques
- Students practice the techniques
- Solutions are corrected (by the teacher)
- Homework is set

Japanese Lesson Study and Lesson Structure

How many squares?





Sullivan Bacalod Plenary 2013

Sullivan Bacalod Plenary 2013

There are Japanese words for parts of lessons

- Hatsumon
  - The initial problem
- Kizuki
  - -what you want them to learn
- Kikanjyuski
  - Individual or group work on the problem
- Kikan shido –
  - thoughtful walking around the desks
- Neriage
  - Carefully managed whole class discussion seeking the students' insights
- Matome
  - Teacher summary of the key ideas

A revised lesson structure Lappan et al. 2006

Launch

Explore

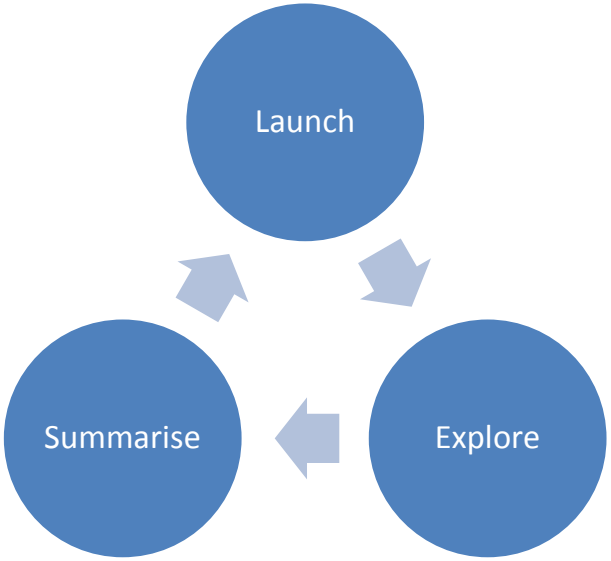
Summarise

Formalising the structure Smith and Stein (2011)

- anticipating potential responses
- monitoring student responses interactively
- selecting representative responses for later presentation
- sequencing student responses
- connecting the students' strategies with the formal processes that were the intention of the task in the first place.

A further revised lesson structure

- In this view, the sequence
  - Launch (without telling)
  - Explore (for themselves)
  - Summarise (drawing on the learning of the students)



- … is cyclical and might happen more than once in a lesson (or learning sequence)

The notion of classroom culture

- Rollard (2012) concluded from the meta analysis that classrooms in which teachers actively support the learning of the students promote high achievement and effort.

Some elements of this active support :

- the identification of tasks that are appropriately challenging for most students;
- the provision of preliminary experiences that are pre-requisite for students to engage with the tasks but which do not detract from the challenge of the task;
- the structuring of lessons including differentiating the experience through the use of enabling and extending prompts for those students who cannot proceed with the task or those who complete the task quickly;

- the potential of consolidating tasks, which are similar in structure and complexity to the original task, with which all students can engage even if they have not been successful on the original task;
- the effective conduct of class reviews which draw on students' solutions to promote discussions of similarities and differences;
- holistic and descriptive forms of assessment that are to some extent self referential for the student and which minimise the competitive aspects; and
- finding a balance between individual thinking time and collaborative group work on tasks.

Getting started

"zone of confusion"

"four before me"

- representing what the task is asking in a different way such as drawing a cartoon or a diagram, rewriting the question …
- choosing a different approach to the task, which includes rereading the question, making a guess at the answer, working backwards …
- asking a peer for a hint on how to get started
- looking at the recent pages in the workbook or textbook for examples.

The lessons consist of

- a challenging task
- a supplementary or consolidating task (see Dooley, 2012)
- preliminary experiences that are pre-requisite but which do not detract from the challenge of the tasks
- supplementary tasks that offer the potential for differentiating the experience through the use of
  - *enabling* prompts (see Sullivan, et al., 2009) which can reduce the number of steps, simplify the complexity of the numbers, and vary the forms of representation for those students who cannot proceed with the task;
  - *extending* prompts for students who complete the original task quickly which often prompt abstraction and generalisation of the solutions.

A probability task

First do this task

- On a plane, the probability that a passenger has a backpack is 0.6, and the probability that a passenger as an MP3 player is 0.7.
- How many passengers might be on the plane?
- How many passengers might have both a backpack and an MP3 player?
- What is the range of possible answers for this?
- Represent each of your solutions in two different ways.

Assume we have 10 people

1	2	3	4	5	6	7	8	9	10
BP	BP	BP	BP	BP	BP				
MP3	MP3	MP3	MP3	MP3	MP3	MP3			

Assume we have 10 people

1	2	3	4	5	6	7	8	9	10
BP	BP	BP	BP	BP	BP				
	MP3	MP3	MP3	MP3	MP3	MP3	MP3		

Assume we have 10 people

1	2	3	4	5	6	7	8	9	10
BP	BP	BP	BP	BP	BP				
		MP3	MP3	MP3	MP3	MP3	MP3	MP3	

Assume we have 10 people

1	2	3	4	5	6	7	8	9	10
BP	BP	BP	BP	BP	BP				
			MP3	MP3	MP3	MP3	MP3	MP3	MP3

PROBABILITIES AND SETS

On a train, the probability that a passenger has a backpack is 0.6, and the probability that a passenger has an MP3 player is 0.7.

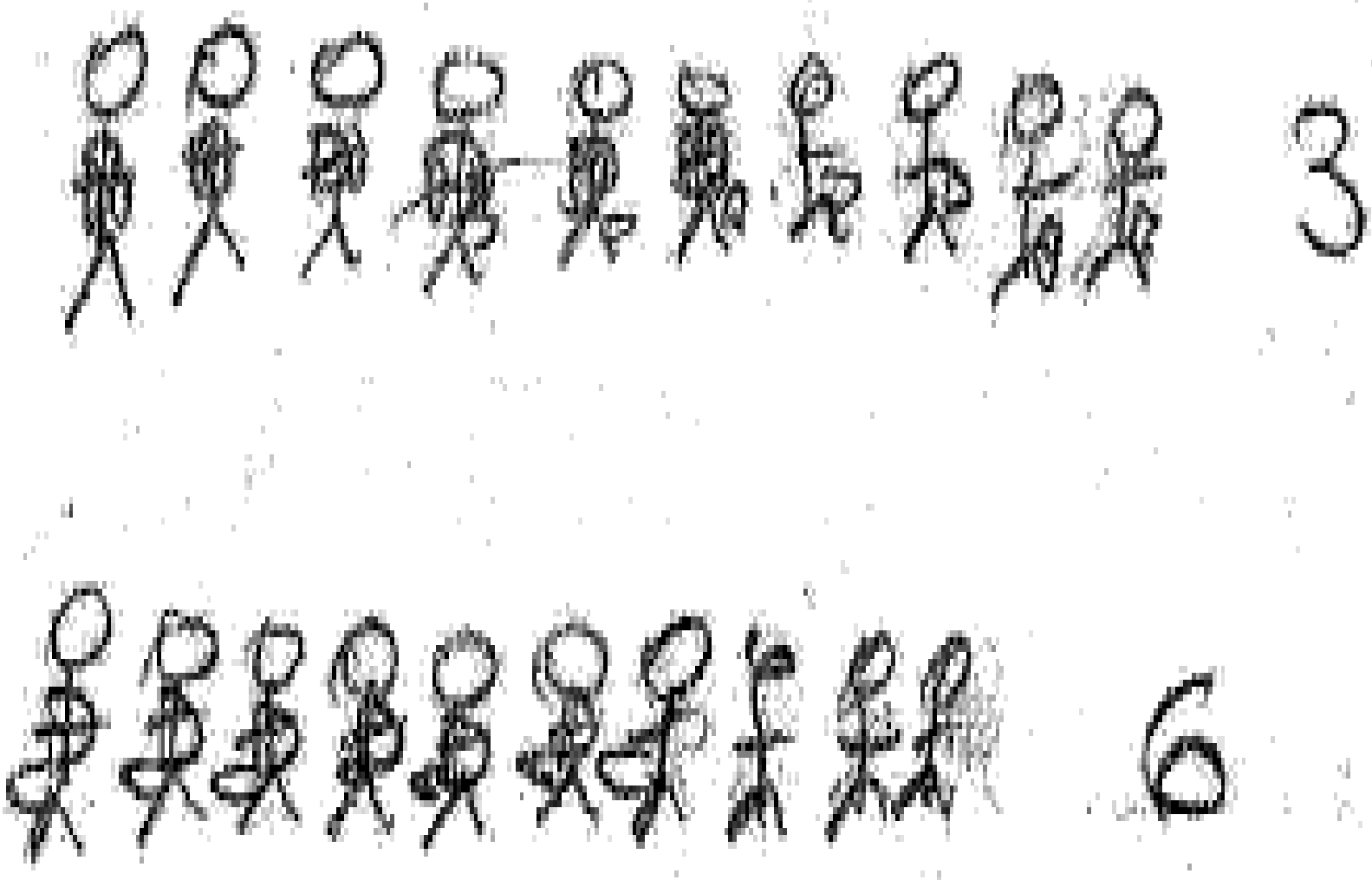
How many passengers might be on the train? 10

How many passengers might have both a backpack and an MP3 player? 0

What is the range of possible answers for this? 3

3

6

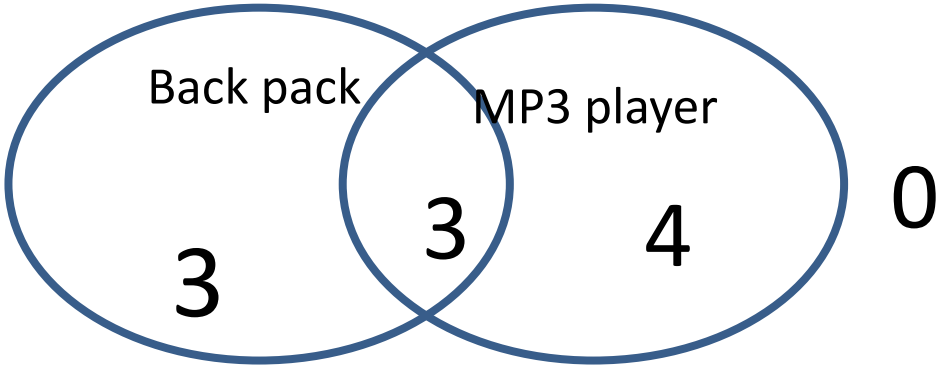

3
6

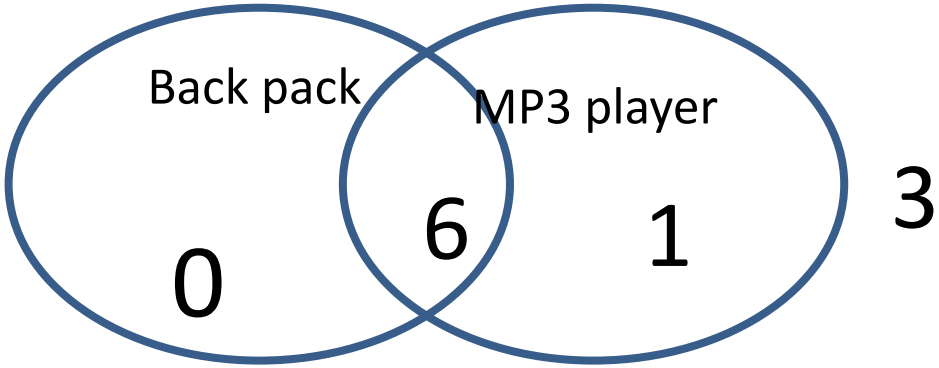
Two way tables

	back pack	No back pack
MP3 player	3	4
No MP3 player	3	0

	back pack	No back pack
MP3 player	6	1
No MP3 player	0	3

Venn diagrams





A consolidating task

- On a plane, the probability that a passenger has a backpack is 0.65, and the probability that a passenger as an MP3 player is 0.57.
- How many passengers might be on the plane?
- What is the maximum and minimum number of possibilities for people who have both a backpack and an MP3 player?
- Represent each of your solutions in two different ways.

An enabling prompt

- On a plane, there are 10 people.
- Six of the people have a backpack, and 7 of the people have an MP3 player.
- How many people might have both a backpack and an MP3 player?
- What is the smallest possible answer for this?
- What is the largest possible answer?

An extending prompt

- On a plane, the probability that a passenger has a backpack is 2/3, and the probability that a passenger has an MP3 player is 2/7.How many passengers might be on the plane? How many passengers might have both a backpack and an MP3 player? What is the range of possible answers for this?
- Represent each of your solutions in two different ways.

The Structure of the Curriculum

ACARA Australian Curriculum, Assessment and Reporting Authority

Contact us | Feedback | User guide | Site map | A A

The Australian Curriculum

Search

acara AUSTRALIAN CURRICULUM ASSESSMENT AND REPORTING AUTHORITY

Home Overview **Curriculum** General capabilities Cross-curriculum priorities FAQs Consultation Download

Mathematics

Statistics and probability
Measurement and geometry
Number and algebra

Rationale/Aims | Organisation | Foundation to Year 10 Curriculum

Filters: Year levels | Strands | General capabilities | Cross-curriculum priorities | Apply filters | Clear filters

View | Show: ☑ Level descriptions ☑ Content descriptions ☑ Achievement standards ☐ Tags | Print this page

F 1 2 3 4 5 6 7 8 9 10 10A

Foundation Year

Foundation Year Level Description

The proficiency strands *Understanding, Fluency, Problem Solving* and *Reasoning* are an integral part of mathematics content across the three content strands: *Number and Algebra*, *Measurement and Geometry*, and *Statistics and Probability*. The proficiencies reinforce the significance of working mathematically within the content and describe how the content is explored or developed. They provide the...

Read full description

Foundation Year Content Descriptions

Number and Algebra	Measurement and Geometry	Statistics and Probability
Number and place value	Using units of measurement	Data representation and interpretation
Establish understanding of the language and processes of counting by naming numbers in sequences, initially to and from 20, moving from any starting point (ACMNA001)	Use direct and indirect comparisons to decide which is longer, heavier or holds more, and explain reasoning in everyday language (ACMMG006)	Answer yes/no questions to collect information (ACMSP011)
Connect number names, numerals and quantities,	Compare and order the duration of events using the everyday language of time (ACMMG007)	

Mathematics

Statistics and probability
Measurement and geometry
Number and algebra

Rationale/Aims | Organisation | Foundation to Year 10 Curriculum

Filters: Showing selected year levels | Strands | General capabilities | Cross-curriculum priorities | Apply filters | Clear filters

View: ions ☑ Content descriptions ☑ Achievement standards ☑ Icons Print this page

4 5 6

Year 4

Year 4 Level Description

The proficiency strands *Understanding, Fluency, Problem Solving and Reasoning* are an integral part of mathematics content across the three content strands: *Number and Algebra, Measurement and Geometry, and Statistics and Probability*. The proficiencies reinforce the significance of working mathematically within the content and describe how the content is explored or developed. They provide the language to build in the developmental aspects of the learning of mathematics.

At this year level:

Understanding includes making connections between representations of numbers, partitioning and combining numbers flexibly, extending place value to decimals, using appropriate language to communicate times, and describing properties of symmetrical

Year 5

Year 5 Level Description

The proficiency strands *Understanding, Fluency, Problem Solving and Reasoning* are an integral part of mathematics content across the three content strands: *Number and Algebra, Measurement and Geometry, and Statistics and Probability*. The proficiencies reinforce the significance of working mathematically within the content and describe how the content is explored or developed. They provide the language to build in the developmental aspects of the learning of mathematics.

At this year level:

Understanding includes making connections between representations of numbers, using fractions to represent probabilities, comparing and ordering fractions and decimals and representing them in various ways, describing transformations and

Year 6

Year 6 Level Description

The proficiency strands *Understanding, Fluency, Problem Solving and Reasoning* are an integral part of mathematics content across the three content strands: *Number and Algebra, Measurement and Geometry, and Statistics and Probability*. The proficiencies reinforce the significance of working mathematically within the content and describe how the content is explored or developed. They provide the language to build in the developmental aspects of the learning of mathematics.

At this year level:

Understanding includes describing properties of different sets of numbers, using fractions and decimals to describe probabilities, representing fractions and decimals in various ways and describing connections between them, and making reasonable estimations

http://www.australiancurriculum.edu.au/Mathematics/Curriculum/F-10?y=4&y=5&y=6&s=NA&s=MG&s=SP&c=2&layout=2

Year 4

Money and financial mathematics

Solve problems involving purchases and the calculation of change to the nearest five cents with and without digital technologies (ACMNA080)

Patterns and algebra

Explore and describe number patterns resulting from performing multiplication (ACMNA081)

Solve word problems by using number sentences involving multiplication or division where there is no remainder (ACMNA082)

Use equivalent number sentences involving addition and subtraction to find unknown quantities (ACMNA083)

Measurement and Geometry

Using units of measurement

Use scaled instruments to measure and compare lengths, masses, capacities and temperatures

Year 5

Money and financial mathematics

Create simple financial plans (ACMNA106)

Patterns and algebra

Use equivalent number sentences involving multiplication and division to find unknown quantities (ACMNA121)

Measurement and Geometry

Using units of measurement

Choose appropriate units of measurement for length, area, volume, capacity and mass

Year 6

Make connections between equivalent fractions, decimals and percentages (ACMNA131)

Money and financial mathematics

Investigate and calculate percentage discounts of 10%, 25% and 50% on sale items, with and without digital technologies (ACMNA132)

Patterns and algebra

Measurement and Geometry

Using units of measurement

Solve problems involving the comparison of leng and areas using appropriate units (ACMMG137)

http://www.australiancurriculum.edu.au/Mathematics/Curriculum/F-10?y=4&y=5&y=6&s=NA&s=MG&s=SP&c=2&layout=2

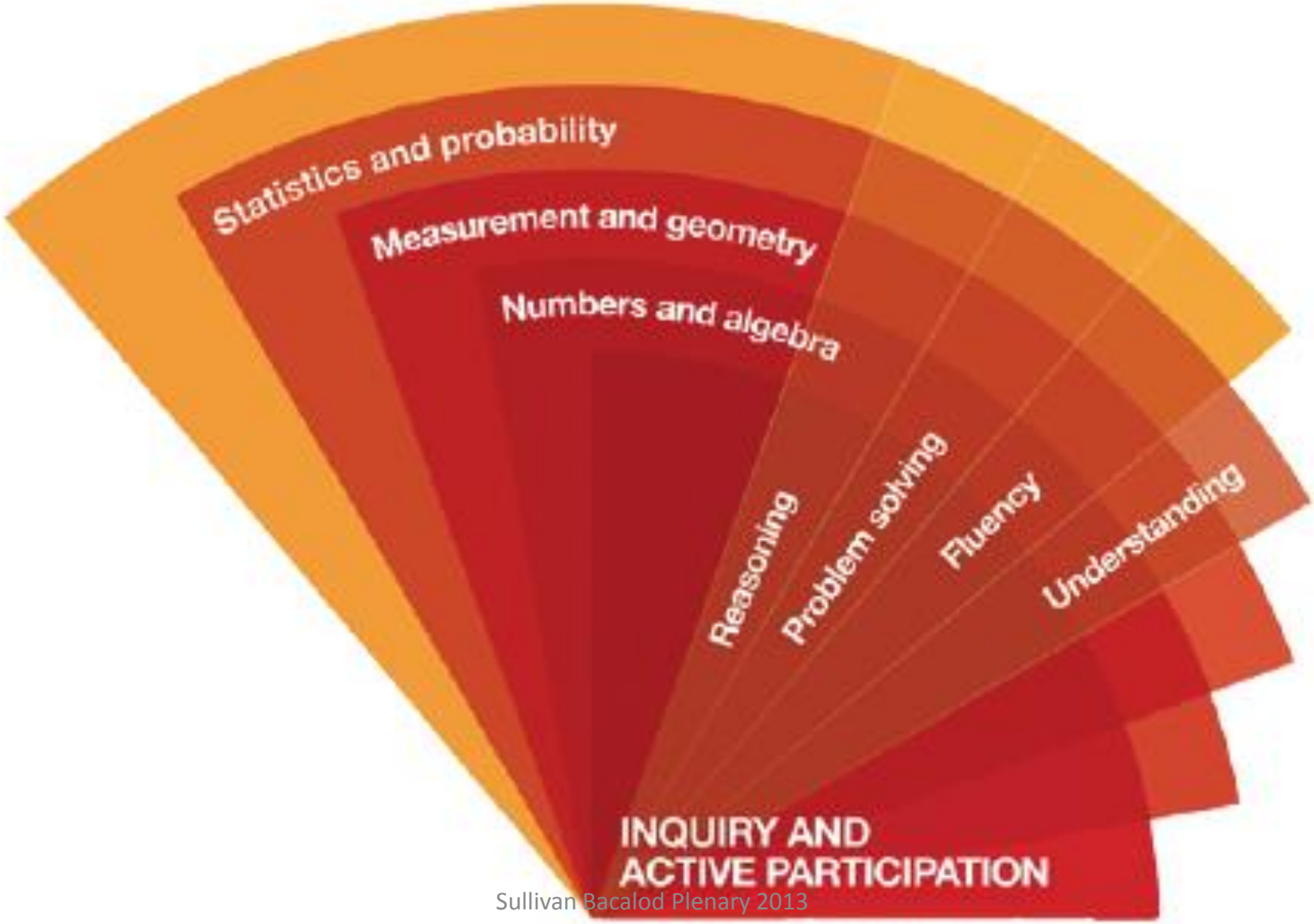
Three Content Strands (nouns)

- Number and Algebra
- Measurement and Geometry
- Statistics and Probability

The content description for this task

- **Represent events in two-way tables and Venn diagrams and solve related problems (ACMSP292)**

The process strands (verbs)



In the Australian curriculum

- Understanding
  - (connecting, representing, identifying, describing, interpreting, sorting, …)
- Fluency
  - (calculating, recognising, choosing, recalling, manipulating, …)
- Problem solving
  - (applying, designing, planning, checking, imagining, …)
- Reasoning
  - (explaining, justifying, comparing and contrasting, inferring, deducing, proving, …)

Some findings from our project

Prompt	Mean score
The level of challenge was about right	4.20
Most students persisted enough to engage with the challenge	4.30
The lesson outline provided all the information I needed	4.50
I would use this lesson again next year (even if I adapt it a little)	4.70
I explained the mathematical purpose to the students	3.85
Most students learned the main mathematical ideas	4.10
The lesson was easy to teach	4.35
The contribution of students to the discussion was good	4.35
This lesson was fine for my best students but most of the others gave up quickly	2.35
I needed to show my students how to do this task,	2.60

	This aspect is important and is needed	I read this aspect carefully when planning
The lesson rationale and the explanation of the mathematical purpose	4.76	4.51
The suggestions about pedagogy	3.67	4.24
The enabling prompts	4.95	4.71
The extending prompts	4.87	4.62
The consolidating tasks	4.65	4.47

17. I know I have between 15 and 25 apples. When they are put into groups of 6 there are 2 apples left over. How many apples do I have?

ANSWER	PRE -TEST	POST- TEST
It is not possible to tell		
18		
20		
22		
Total		

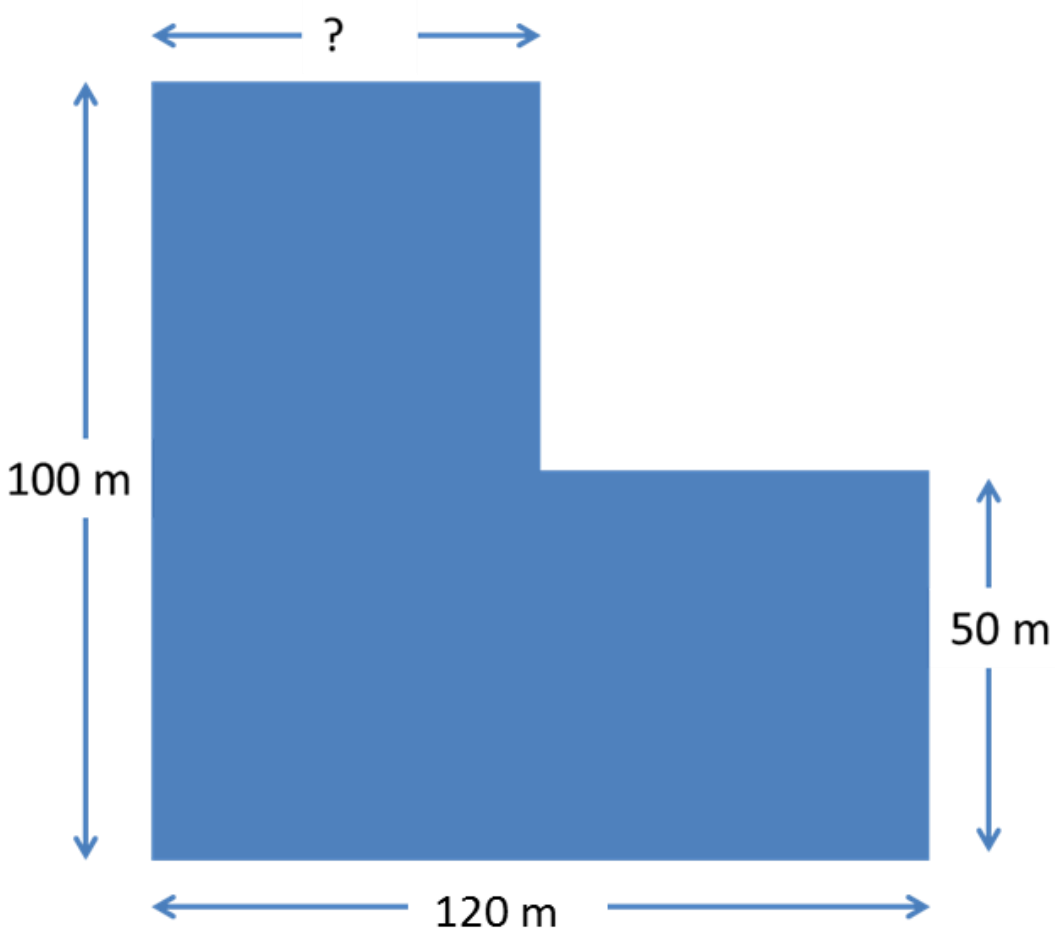
	PRE- TEST	POST -TEST
I prefer tasks I work on in class to be much harder than the apples question	29%	34%
I prefer tasks I work on in class to be about as hard as the apples question	56%	55%
I prefer tasks I work on in class to be much easier than the apples question	15%	12%
TOTAL	100%	100%

	PRE - TEST	POST - TEST
I prefer to work out tasks like the apples question for myself	39%	41%
I prefer to work on tasks like the apples question with other students	44%	47%
I prefer to be told what to do by the teacher	17%	12%
TOTAL	100%	100%

ALL RESPONSES (POST -TEST)

	I prefer to work on tasks like the apples question for myself	I prefer to work on tasks like the apples question with some other students	I prefer to be told what to do by the teacher	
I prefer tasks I work on in class to be much harder than the apples question	180	90	25	295
I prefer tasks I work on in class to be about as hard as the apples question	175	238	56	469
I prefer tasks I work on in class to be much easier than the apples question	21	52	26	99
Total	376	380	107	863

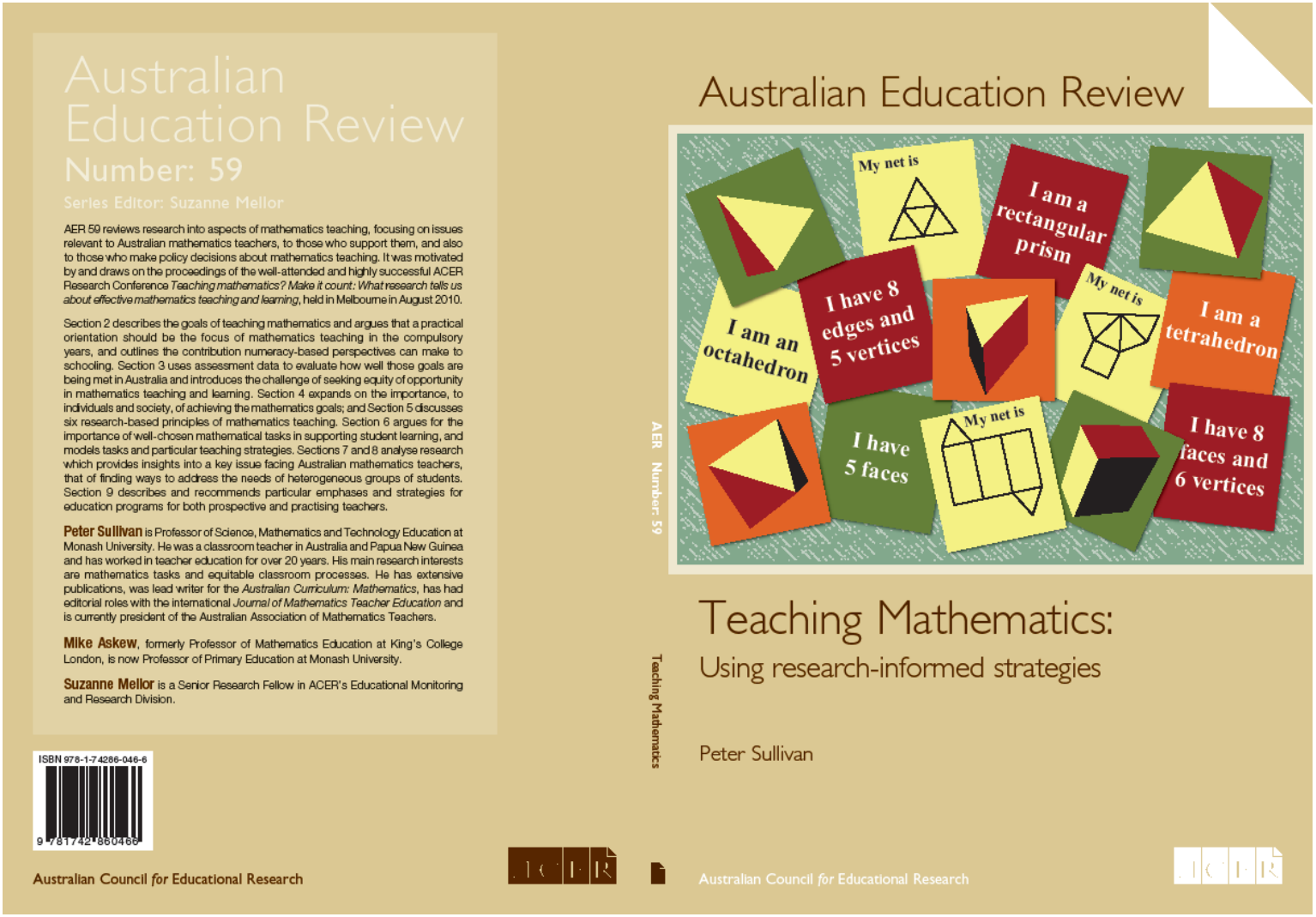
This is a paddock in the shape of an L. The area is 1 hectare. How many metres wide is the top part of the L? (A square 100 m x 100 m has an area of 1 hectare) (Diagram not drawn to scale)



Answer	Pre-Test	Post-Test
I prefer tasks I work on in class to be much harder than the paddock question	16%	16%
I prefer tasks I work on in class to be about as hard as the paddock question	62%	61%
I prefer tasks I work on in class to be much easier than the paddock question	22%	23%
	100%	100%

Answer	Pre-Test	Post-Test
I prefer to work out tasks like the paddock question for myself	31%	32%
I prefer to work on tasks like the paddock question with other students	54%	58%
I prefer to be told what to do by the teacher	15%	11%
	100%	100%


EMERGENCY
174 KM AHEAD



Australian Education Review

Number: 59

Series Editor: Suzanne Mellor

AER 59 reviews research into aspects of mathematics teaching, focusing on issues relevant to Australian mathematics teachers, to those who support them, and also to those who make policy decisions about mathematics teaching. It was motivated by and draws on the proceedings of the well-attended and highly successful ACER Research Conference *Teaching mathematics? Make it count: What research tells us about effective mathematics teaching and learning*, held in Melbourne in August 2010.

Section 2 describes the goals of teaching mathematics and argues that a practical orientation should be the focus of mathematics teaching in the compulsory years, and outlines the contribution numeracy-based perspectives can make to schooling. Section 3 uses assessment data to evaluate how well those goals are being met in Australia and introduces the challenge of seeking equity of opportunity in mathematics teaching and learning. Section 4 expands on the importance, to individuals and society, of achieving the mathematics goals; and Section 5 discusses six research-based principles of mathematics teaching. Section 6 argues for the importance of well-chosen mathematical tasks in supporting student learning, and models tasks and particular teaching strategies. Sections 7 and 8 analyse research which provides insights into a key issue facing Australian mathematics teachers, that of finding ways to address the needs of heterogeneous groups of students. Section 9 describes and recommends particular emphases and strategies for education programs for both prospective and practising teachers.

Peter Sullivan is Professor of Science, Mathematics and Technology Education at Monash University. He was a classroom teacher in Australia and Papua New Guinea and has worked in teacher education for over 20 years. His main research interests are mathematics tasks and equitable classroom processes. He has extensive publications, was lead writer for the *Australian Curriculum: Mathematics*, has had editorial roles with the international *Journal of Mathematics Teacher Education* and is currently president of the Australian Association of Mathematics Teachers.

Mike Askew, formerly Professor of Mathematics Education at King's College London, is now Professor of Primary Education at Monash University.

Suzanne Mellor is a Senior Research Fellow in ACER's Educational Monitoring and Research Division.

ISBN 978-1-74286-046-6

Australian Council *for* Educational Research

AER Number: 59

Teaching Mathematics

Australian Education Review

Teaching Mathematics:

Using research-informed strategies

Peter Sullivan

Australian Council *for* Educational Research

AVAILABLE TO DOWNLOAD FREE FROM

http://research.acer.edu.au/aer/13/ aer